Statistical Power: Foundations and Applications

Design-specific and General Approaches to Power and Sample Size Analyses

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Family Studies Center Presentation

Presentation Goals

- Statistical Power Analysis
  - Background, context, and definition of Statistical Power Analysis
  - Statistical foundations of Power Analysis
  - Power analysis software
- Doing Power Analysis for Standard Designs
  - Difference Between two Means
  - Multiple Linear Regression Increment in R^2
  - Difference Between Two Correlations
  - Comparing Two Means with Clustered Data
- Power Analysis for Other Designs Using Web Apps and Simulation
  - Structural Equation Models
  - Latent Growth Models
  - Actor-Partner Interdependence Model (APIM)
  - Mediation Model
  - Extended Mediation Analysis

Why Pay Attention to Statistical Power? Because You Want To…

- Statistically detect effects that are
  - Scientifically important
  - Clinically meaningful
  - More publishable
- Allocate sufficient but not excessive ***resources*** to data collection
- Provide information to grant reviewers, IRB, etc. to assess the proposal's quality, ***potential for success***, and funding worthiness
- Contribute reliable and ***reproducible results*** to the literature

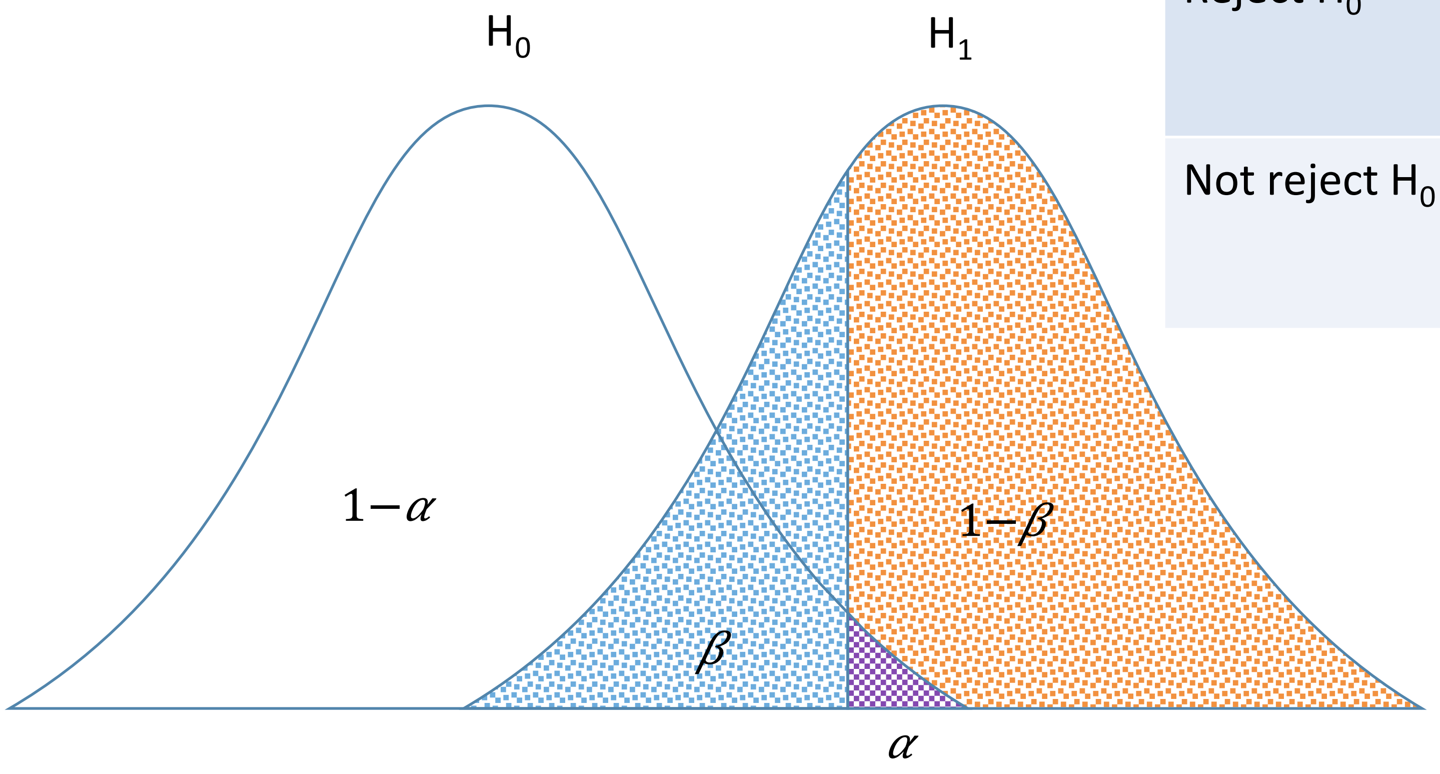
What are the Consequences of Insufficient Power?

- Recent reviews have found statistical power to detect the typical effect size in social/personality psychology research of approximately 50% with average power in some other fields as low as 20% - 30%
  - Underpowered studies are ***less able to detect true relationships***
  - Low power studies lead to a ***greater proportion of false positives*** in the broader literature
  - The findings of low power original studies are ***less likely to be successfully reproduced*** through either direct or conceptual replication

When do Researchers Report Power Analyses?

- Reviews have found that only about 3 - 5% of research articles in psychological journals report power analyses (excluding theoretical, qualitative, single-case, simulation studies, meta-analyses), but this may have improved to as much as 10 - 15% in the last few years.
  - Researchers are more likely to report power analyses if journals include prospective power analysis in their ***submission guidelines and editorial recommendations***
    - Nature Medicine 3%
    - New England Journal of Medicine 61%
  - Researchers are more likely to report power analyses for ***data collected during the study than for previously collected data***
    - Sample size determination for newly collected data
    - Minimal effect size determination for existing data
  - Researchers are more likely to report power analyses if they have received ***training*** to conduct power analyses or have ***previous experience*** performing power analyses

What is Statistical Power?



Decision	H_0 True	H_0 False
Reject H_0	Type I Error False Positive $p=\alpha$	Correct Decision True Positive $p=1-\beta$
Not reject H_0	Correct Decision True Negative $p=1-\alpha$	Type II Error False Negative $p=\beta$

Power

Power and statistical decision making

- **Type I error (α: False positive)**
  - Probability of incorrectly rejecting a true null hypothesis
- **Type II error (β: False negative)**
  - Probability of incorrectly failing to reject a false null hypothesis
- **Power ($1-\beta$: True positive)**
  - Probability of correctly rejecting a false null hypothesis
- **True negative ($1-\alpha$)**
  - Probability of correctly failing to reject a true null hypothesis

Decision	H_0 True	H_0 False	
Reject H_0	Type I Error False Positive α	Correct Decision True Positive $1-\beta$	Power
Not reject H_0	Correct Decision True Negative $1-\alpha$	Type II Error False Negative β	

They all go together

- **Sample size** (a count: N)
  - How many people do I need in order to have a good chance of finding the effect?
- **Power** (a probability: $1-\beta$)
  - What's the chance of finding an effect that really is there?
- **Effect size** (a value: δ)
  - How big does an effect have to be in order to find it?
- **Significance level—Type I error** (a probability: α)
  - What's the chance of claiming an effect that doesn't really exist?
- **Type II error** (a probability: β)
  - What's the chance of missing a real effect?
- **[Residual] Standard deviation** (a value: S)
  - How much variability is there in the outcome [after accounting for the effect]?

Some Common uses of Statistical Power Analysis

- Before the data is collected
  - Determine the ***sample size*** (N) needed to achieve a given level of power ($1-\beta$) for a designated effect size (δ) and significance level (α)
- After the data is collected but before it is analyzed
  - Determine the ***expected level of power*** ($1-\beta$) given the obtained sample size (N) for a designated effect size (δ) and significance level (α)
  - Determine the ***minimum detectable effect size*** (δ) given the obtained sample size (N) for a given level of power ($1-\beta$) at a designated significance level (α)

Statistical Foundations of Power Analyses

The Centrality of Noncentrality

- The traditional Z ***test statistic***:
  - $Z = \frac{y_1 - y_2}{\sigma\sqrt{1/n_1 + 1/n_2}}$
- The ***noncentrality parameter*** as the expectation of the test statistic where there is a mean difference in the population:
  - $\lambda = E\left(\frac{y_1 - y_2}{\sigma\sqrt{1/n_1 + 1/n_2}}\right) = \frac{\mu_1 - \mu_2}{\sigma}\sqrt{\frac{n_1 n_2}{n_1 + n_2}} = \sqrt{\frac{n_1 n_2}{n_1 + n_2}}\,(\Delta/\sigma)$
- A simplification with ***equal group sizes*** $(n_1 = n_2 = n_c)$:
  - $\lambda = \sqrt{\frac{n_c}{2}}\,(\Delta/\sigma)$

Power for Mean Difference Testing

- Power for ***one-sided hypotheses*** where Φ is the cumulative normal distribution function and α is designated *significance level* for the test:
  - $1-\beta_{(one-sided)} = \Phi(\lambda - z_{1-\alpha})$
- Power for ***two-sided hypotheses***
  - $1-\beta_{(two-sided)} = \Phi(\lambda - z_{1-\alpha/2}) + \Phi(z_{1-\alpha/2} - \lambda)$

Sample Size Determination

- The standard normal ***power quantile*** for $(1-\beta)$:
  - $z_{1-\beta} = \lambda - z_{1-\alpha} = \sqrt{n_c/2}\,(\Delta/\sigma) - z_{1-\alpha}$
- Solving for the common group ***sample size* for *one-sided*** hypotheses
  - $n_c = 2\sigma^2 \left(\frac{z_{1-\beta} + z_{1-\alpha}}{\Delta}\right)^2$
- Approximating the ***sample size* for *two-sided*** hypotheses
  - $n_c \approx 2\sigma^2 \left(\frac{z_{1-\beta} + z_{1-\alpha/2}}{\Delta}\right)^2$

Effect Size: Simple, Standardized, and Minimum Detectable

- The ***simple effect size***
  - $\mu_1 - \mu_2 = \Delta$
- The ***standardized effect size*** (aka Cohen's d):
  - $d = \Delta / \sigma$
- Approximating the ***minimum detectable effect size*** for a given sample size n_c and power $(1-\beta)$ for ***two-sided*** hypotheses:
  - $\Delta \approx \sqrt{2/n_c}\ \sigma(z_{1-\beta} + z_{1-\alpha/2})$

Common Conventions for Effect Size, Power, and Statistical Significance Level

- Standardized effect sizes: Cohen's d (.2, .5, .8)
- Power: $1-\beta$ (.80, .90, .95)
- Significance level: α (.001, .01, .05)

Software for Power Analysis and Sample Size Determination

Commercial Statistical Power Analysis Software

- Stata power
  - http://www.stata-press.com/manuals/power-sample-size-reference-manual/
- SAS Proc Power & Proc GLMPower
  - https://support.sas.com/documentation/onlinedoc/stat/131/power.pdf
  - https://support.sas.com/documentation/onlinedoc/stat/131/glmpower.pdf
- NCSS Sample Size and Power (PASS)
  - https://www.ncss.com/software/pass/
  - https://www.ncss.com/software/pass/procedures/
- Power and Precision
  - http://www.power-analysis.com/
- Nquery
  - https://www.statsols.com/nquery-sample-size-and-power-calculation-for-successful-clinical-trials

Freely Available Statistical Power Analysis Software

- G*Power (Windows & Mac)
  - http://www.gpower.hhu.de/en.html
- PS: Power and Sample Size Calculation (Windows & Mac)
  - http://biostat.mc.vanderbilt.edu/wiki/Main/PowerSampleSize
- Applets for Power and Sample Size (Java)
  - https://homepage.divms.uiowa.edu/~rlenth/Power/
- R package pwr
  - https://cran.r-project.org/web/packages/pwr/index.html
  - https://www.statmethods.net/stats/power.html
  - http://www.evolutionarystatistics.org/document.pdf
- Optimal Design (Version 3.01) Software for Multi-level and Longitudinal Research (Windows)
  - https://sites.google.com/site/optimaldesignsoftware/

Doing Power Analysis in G*Power and Stata

Difference Between Two Means

Doing the analyses:
Two group mean difference test

- The main rules:
  - You can only ask one question at a time
  - For each question, you must supply answers (or guesses) for the remaining ones
- Additional subtleties:
  - Are the population standard deviations known or unknown?
  - Are the two group sample sizes equal?
  - Are the two group standard deviations the same?

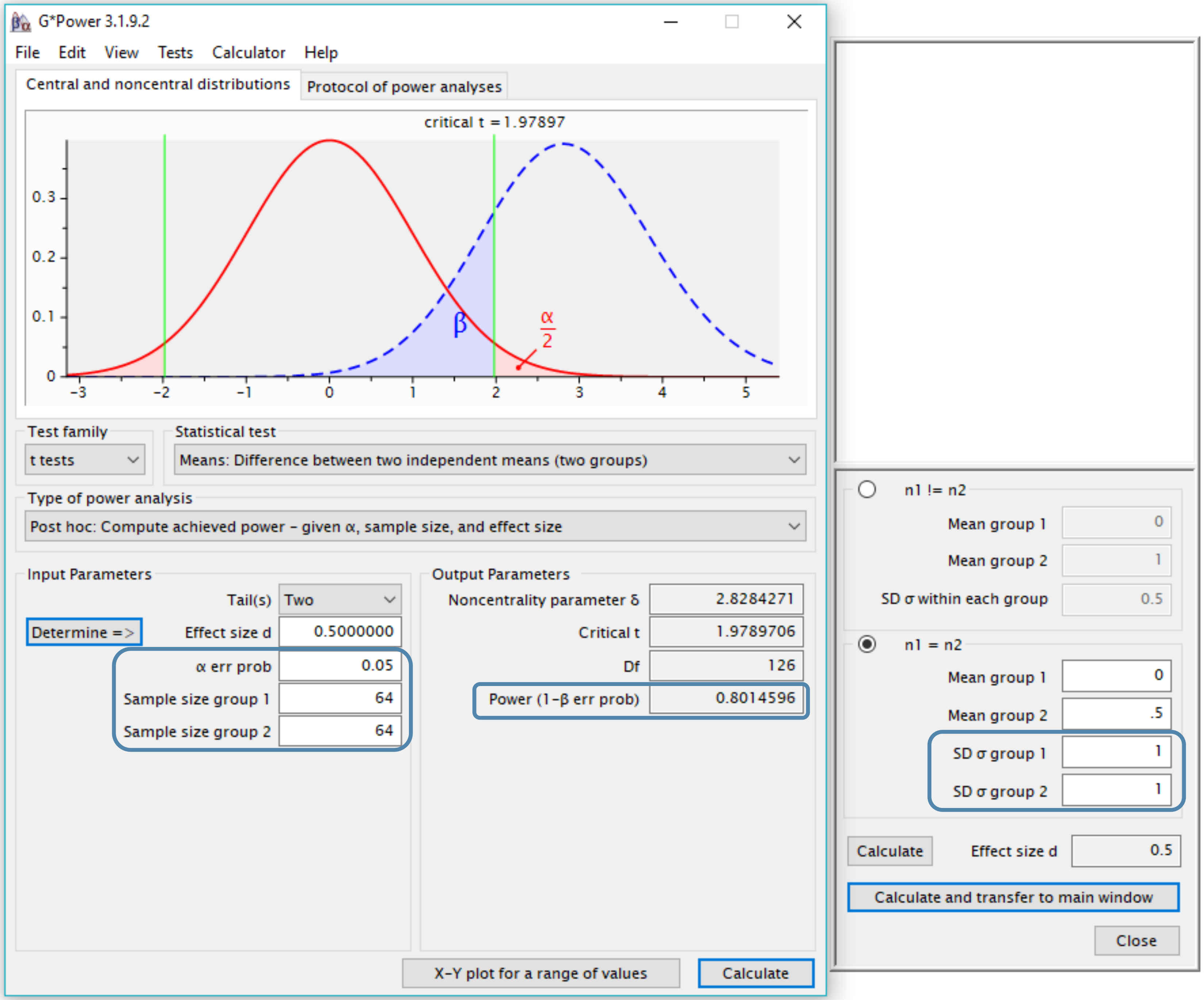
Do it with G*Power: Two group mean difference test

Group 1

$\mu_1 = 0$

Group 2

$\mu_2 = .5$



Do it with Stata:
Two group mean difference test

- **(Total) sample size**
  - `power twomeans 0, diff(.5) power(.8) sd(1) alpha(.05)`
- **Power**
  - `power twomeans 0, diff(.5) n(100) sd(1) alpha(.05)`
- **Minimum detectable effect size**
  - `power twomeans 0, n(100) power(.8) sd(1) alpha(.05)`
- Power with ***unequal group sizes***
  - `power twomeans 0, diff(.5) n1(30) n2(70) sd(1) alpha(.05)`
- Power with ***unequal group standard deviations***
  - `power twomeans 0, diff(.5) n(100) sd1(.8) sd2(1.2) alpha(.05)`

Optional defaults

Estimating Sample size, Power, & Effect size: Two-group mean difference test

		Sample size	Power	Effect size
Significance level	α	.05	.05	.05
Power	$1-\beta$	.80	***.6969***	.80
Sample size	N	***128***	100	100
Effect size	δ	.50	.50	***.5659***
Standard deviation	S	1.00	1.00	1.00

They go together

Examine Sample Size for a Range of Effect Sizes

```
power twomeans 0 (.3(.1).9), alpha(.05) power(.8) sd(1)
```

Optional (default)

```
Performing iteration ...
Estimated sample sizes for a two-sample means test
t test assuming sd1 = sd2 = sd
Ho: m2 = m1  versus  Ha: m2 != m1
```

alpha	power	N	N1	N2	delta	m1	m2	sd
.05	.8	352	176	176	.3	0	.3	1
.05	.8	200	100	100	.4	0	.4	1
.05	.8	128	64	64	.5	0	.5	1
.05	.8	90	45	45	.6	0	.6	1
.05	.8	68	34	34	.7	0	.7	1
.05	.8	52	26	26	.8	0	.8	1
.05	.8	42	21	21	.9	0	.9	1

Default Values (alpha, power)

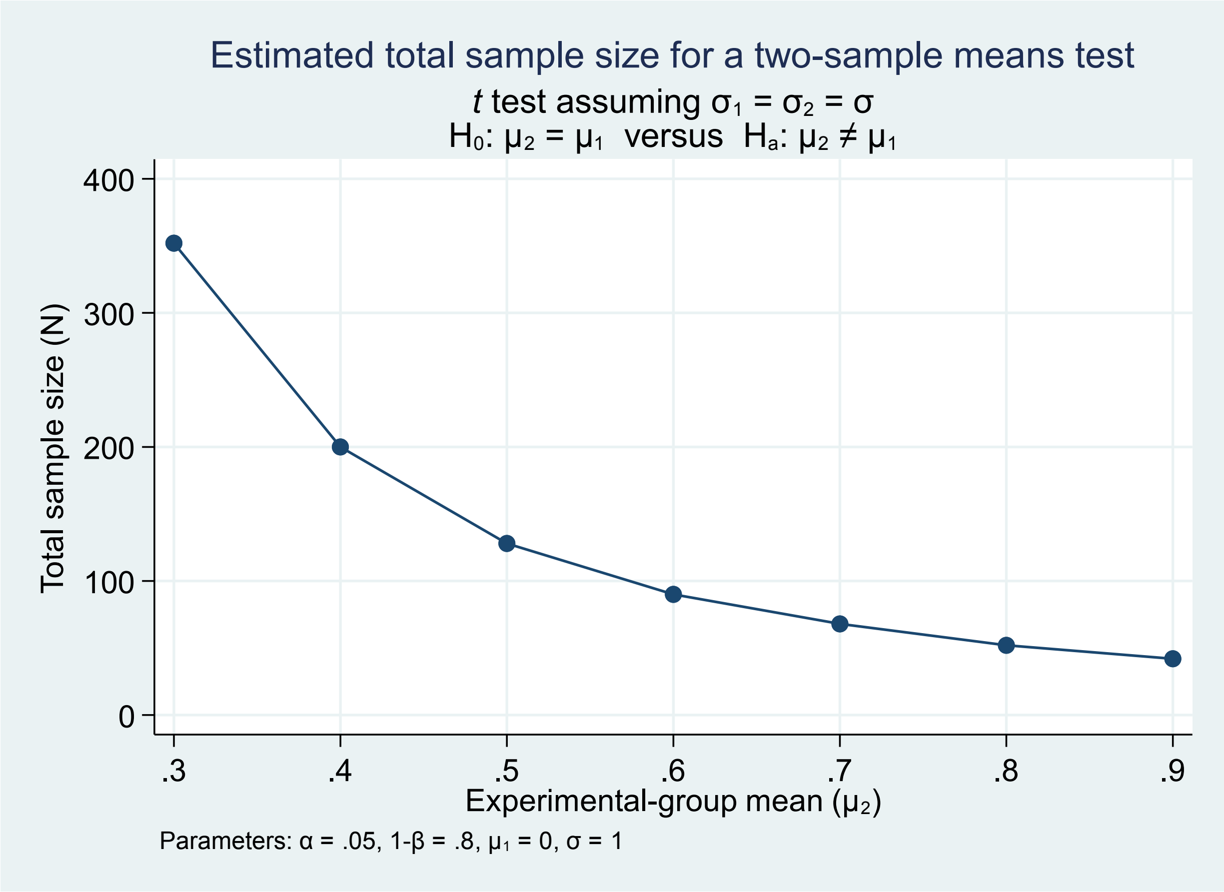
Estimate Sample size (N)

Specify Range of effect sizes (delta, m2)

Default Values (sd)

Plot Sample Size for a Range of Effect Sizes

```
. power twomeans 0 (.3(.1).9), graph
```



Examine Power for a Range of Sample Sizes

```
. power twomeans 0 .5, n(80(10)160) alpha(.05) sd(1)
```

Optional (default)

```
Estimated power for a two-sample means test
t test assuming sd1 = sd2 = sd
Ho: m2 = m1  versus  Ha: m2 != m1
```

alpha	power	N	N1	N2	delta	m1	m2	sd
.05	.5981	80	40	40	.5	0	.5	1
.05	.6502	90	45	45	.5	0	.5	1
.05	.6969	100	50	50	.5	0	.5	1
.05	.7385	110	55	55	.5	0	.5	1
.05	.7753	120	60	60	.5	0	.5	1
.05	.8076	130	65	65	.5	0	.5	1
.05	.8358	140	70	70	.5	0	.5	1
.05	.8604	150	75	75	.5	0	.5	1
.05	.8816	160	80	80	.5	0	.5	1

Default Value

Estimate Power

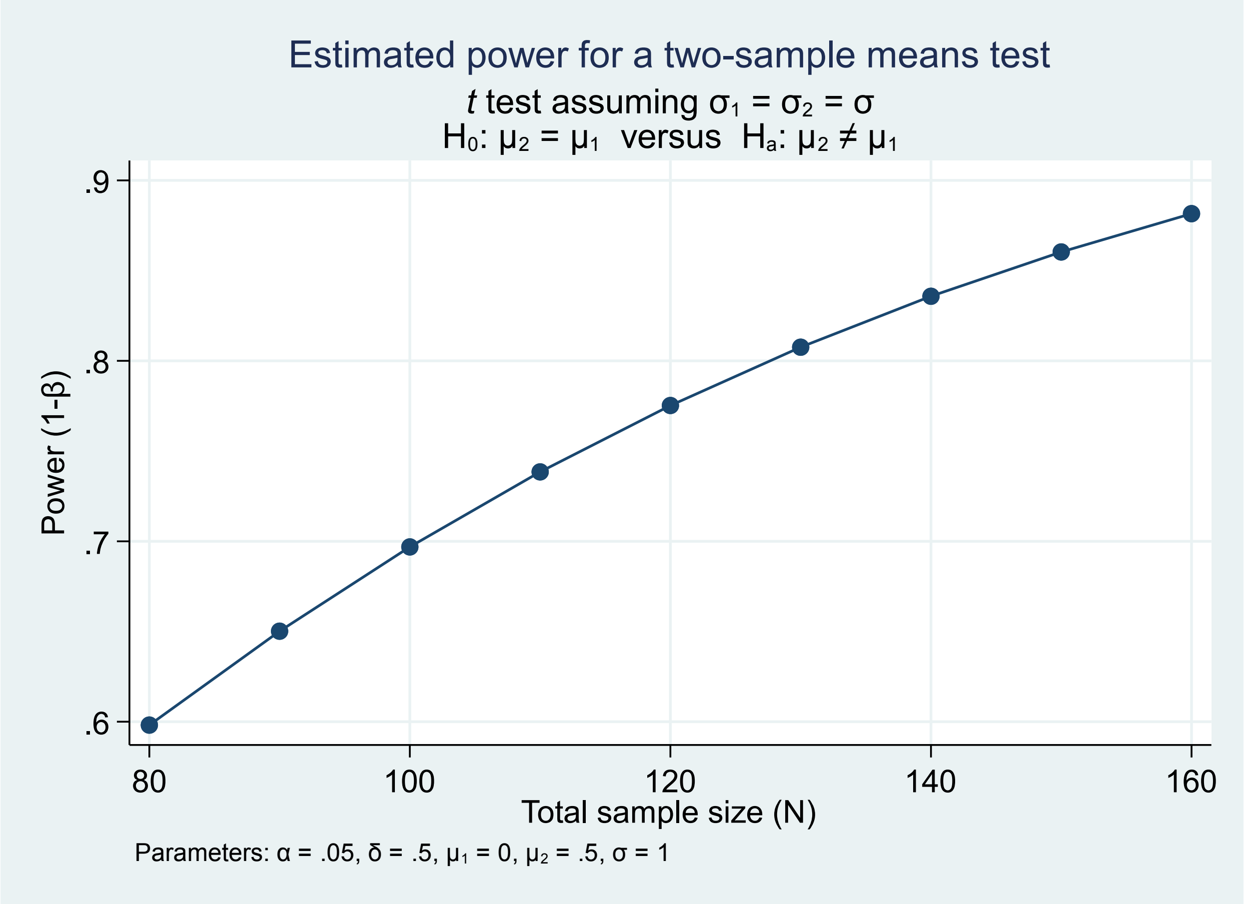
Specify range of sample sizes

Specify effect size

Default Value

Plot Power for a Range of Sample Sizes

```
. power twomeans 0 .5, n(80(10)160) graph
```



Examine the Minimum Detectable Effect Size for a Range of Sample Sizes

```
. power twomeans 0, power(.8) n(80(10)160) alpha(.05) sd(1)
```

Optional (default): alpha(.05) sd(1)

```
Estimated experimental-group mean for a two-sample means test
t test assuming sd1 = sd2 = sd
Ho: m2 = m1  versus  Ha: m2 != m1; m2 > m1
```

alpha	power	N	N1	N2	delta	m1	m2	sd
.05	.8	80	40	40	.6343	0	.6343	1
.05	.8	90	45	45	.5972	0	.5972	1
.05	.8	100	50	50	.5659	0	.5659	1
.05	.8	110	55	55	.5391	0	.5391	1
.05	.8	120	60	60	.5157	0	.5157	1
.05	.8	130	65	65	.4952	0	.4952	1
.05	.8	140	70	70	.4769	0	.4769	1
.05	.8	150	75	75	.4605	0	.4605	1
.05	.8	160	80	80	.4457	0	.4457	1

- alpha: Default
- power: Specify Power
- N: Specify range of sample sizes
- delta, m2: Estimate effect size
- sd: Default

Examine Sample Size for a Selected Range of Significance Levels

```
power twomeans 0 .5, alpha(.001 .01 .05 .1) power(.8) sd(1)
```

Optional (default)

```
Estimated sample sizes for a two-sample means test
t test assuming sd1 = sd2 = sd
Ho: m2 = m1  versus  Ha: m2 != m1
```

alpha	power	N	N1	N2	delta	m1	m2	sd
.001	.8	280	140	140	.5	0	.5	1
.01	.8	192	96	96	.5	0	.5	1
.05	.8	128	64	64	.5	0	.5	1
.1	.8	102	51	51	.5	0	.5	1

Significance Level (alpha) · Power (power) · Sample Size (N) · Effect Size (delta, m2) · Standard Devition (sd)

Paired t-test for Dependent Data

- Research Scenarios
  - Pre-post differences
  - Husband-wife differences
- Statistical Considerations
  - Power and sample size depend on the pre-post correlation ρ
  - Standard error of the difference: $SE_{diff} = \sigma\sqrt{2(1-\rho)}$
  - Standardized effect size: $\delta = (x_2 - x_1)/SE_{diff}$

Examine Sample Size for a Selected Range of Pre-post Correlations

```
power pairedmeans 0 .5, corr(.2(.1).8) alpha(.05) power(.8) sd(1)
```

Optional (default)

```
Estimated sample size for a two-sample paired-means test
Paired t test assuming sd1 = sd2 = sd
Ho: d = d0  versus  Ha: d != d0
```

Default

alpha	power	N	delta	d0	da	ma1	ma2	sd_d	sd	corr
.05	.8	53	.3953	0	.5	0	.5	1.265	1	.2
.05	.8	46	.4226	0	.5	0	.5	1.183	1	.3
.05	.8	40	.4564	0	.5	0	.5	1.095	1	.4
.05	.8	34	.5	0	.5	0	.5	1	1	.5
.05	.8	28	.559	0	.5	0	.5	.8944	1	.6
.05	.8	21	.6455	0	.5	0	.5	.7746	1	.7
.05	.8	15	.7906	0	.5	0	.5	.6325	1	.8

Default values

Estimate sample size

Standardized effect size

Unstandardize d effect size

Standard error of the difference

Specify range of correlations

Doing Power Analysis in Stata

Multiple Linear Regression Increment in R^2 Power Analyses

Examine Sample Size for a Specified Range of Power and the Number of Tested and Controlled Variables

```
power rsquared .3 .4, power(.7(.05).9) ntested(2) ncontrol(3) alpha(.05)
```

```
Estimated sample size for multiple linear regression
F test for R2 testing subset of coefficients
Ho: R2_F = R2_R  versus  Ha: R2_F != R2_R
```

alpha	power	N	delta	R2_R	R2_F	R2_diff	ntested	ncontrol
.05	.7	50	.1667	.3	.4	.1	2	3
.05	.75	55	.1667	.3	.4	.1	2	3
.05	.8	62	.1667	.3	.4	.1	2	3
.05	.85	69	.1667	.3	.4	.1	2	3
.05	.9	80	.1667	.3	.4	.1	2	3

Default values

Specify power

Estimate sample size

Std. eff. size

R^2 values and change in R^2

Number of tested and control variables

Examine Power for a Specified Range of Sample Sizes and the Number of Tested and Controlled Variables

```
power rsquared .3 .4, n(40(10)80) ntested(2) ncontrol(3) alpha(.05)
```

Optional (default)

```
Estimated power for multiple linear regression
F test for R2 testing subset of coefficients
Ho: R2_F = R2_R  versus  Ha: R2_F != R2_R
```

alpha	power	N	delta	R2_R	R2_F	R2_diff	ntested	ncontrol
.05	.5916	40	.1667	.3	.4	.1	2	3
.05	.7047	50	.1667	.3	.4	.1	2	3
.05	.792	60	.1667	.3	.4	.1	2	3
.05	.8569	70	.1667	.3	.4	.1	2	3
.05	.9034	80	.1667	.3	.4	.1	2	3

Default values

Estimate power

Sample sizes

Std. eff. size

Restricted and Full R^2 values

Change in R^2

Number of tested and control variables

Doing Power Analysis with G*Power

Difference Between Two Correlations

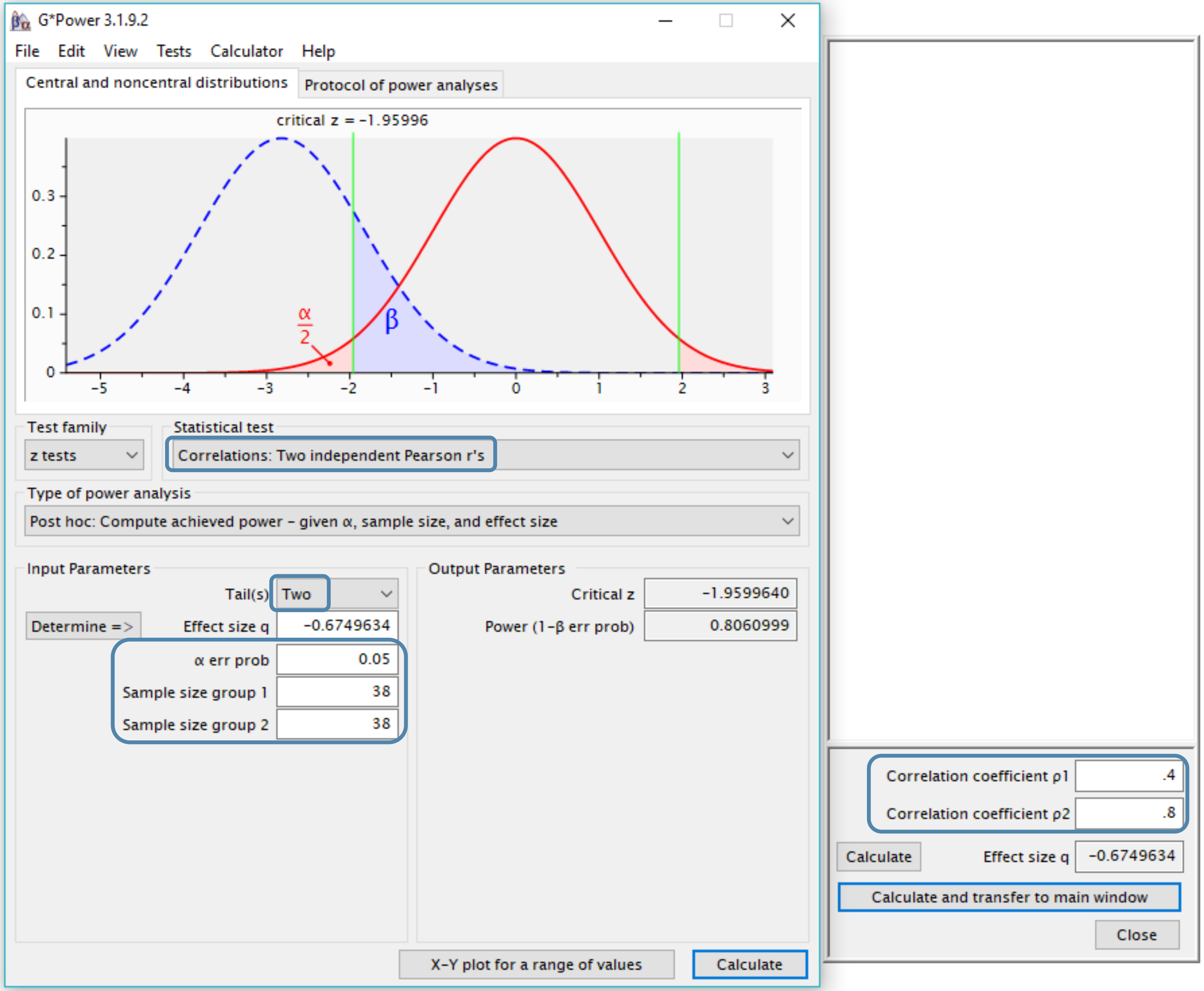
Difference Between Two Independent Correlations

Group 1

		Perf a	Mem b
Perf	a	1.00	.40
Mem	b		1.00

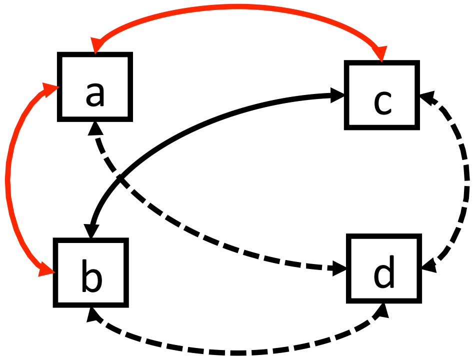
Group 2

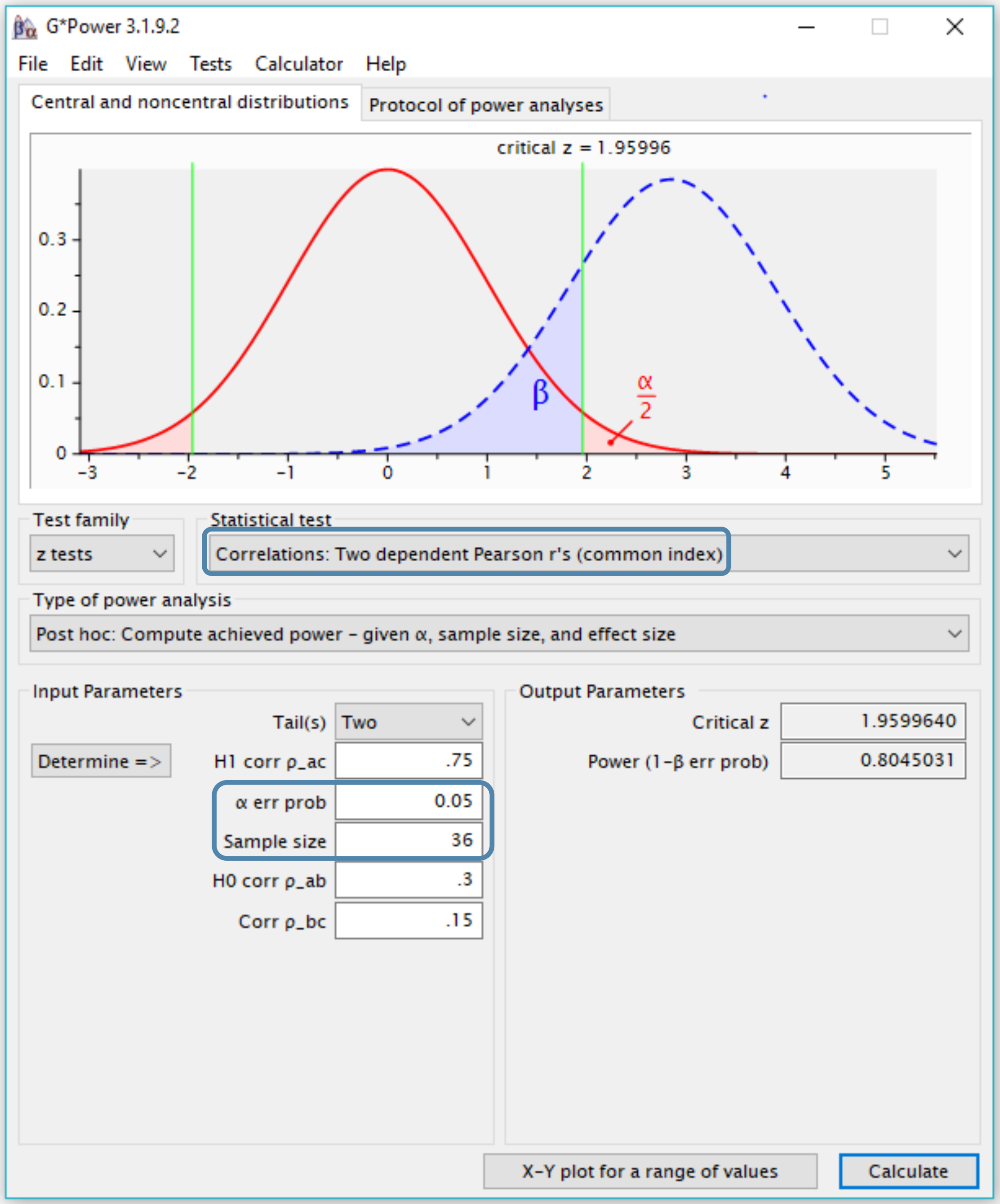
		Perf a	Mem b
Perf	a	1.00	.80
Mem	b		1.00



Difference Between Two Overlapping Dependent Correlations

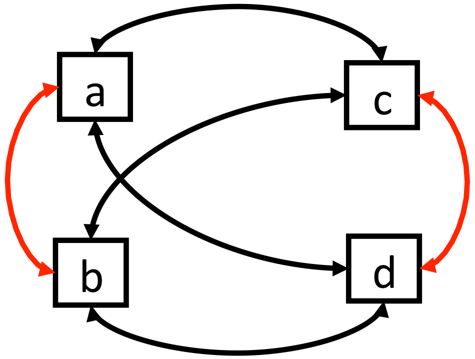
			Pre		Post	
			Perf	Mem	Perf	Mem
			a	b	c	d
Pre	Perf	a	1.00	.30	.75	.25
	Mem	b		1.00	.15	.65
Post	Perf	c			1.00	.50
	Mem	d				1.00

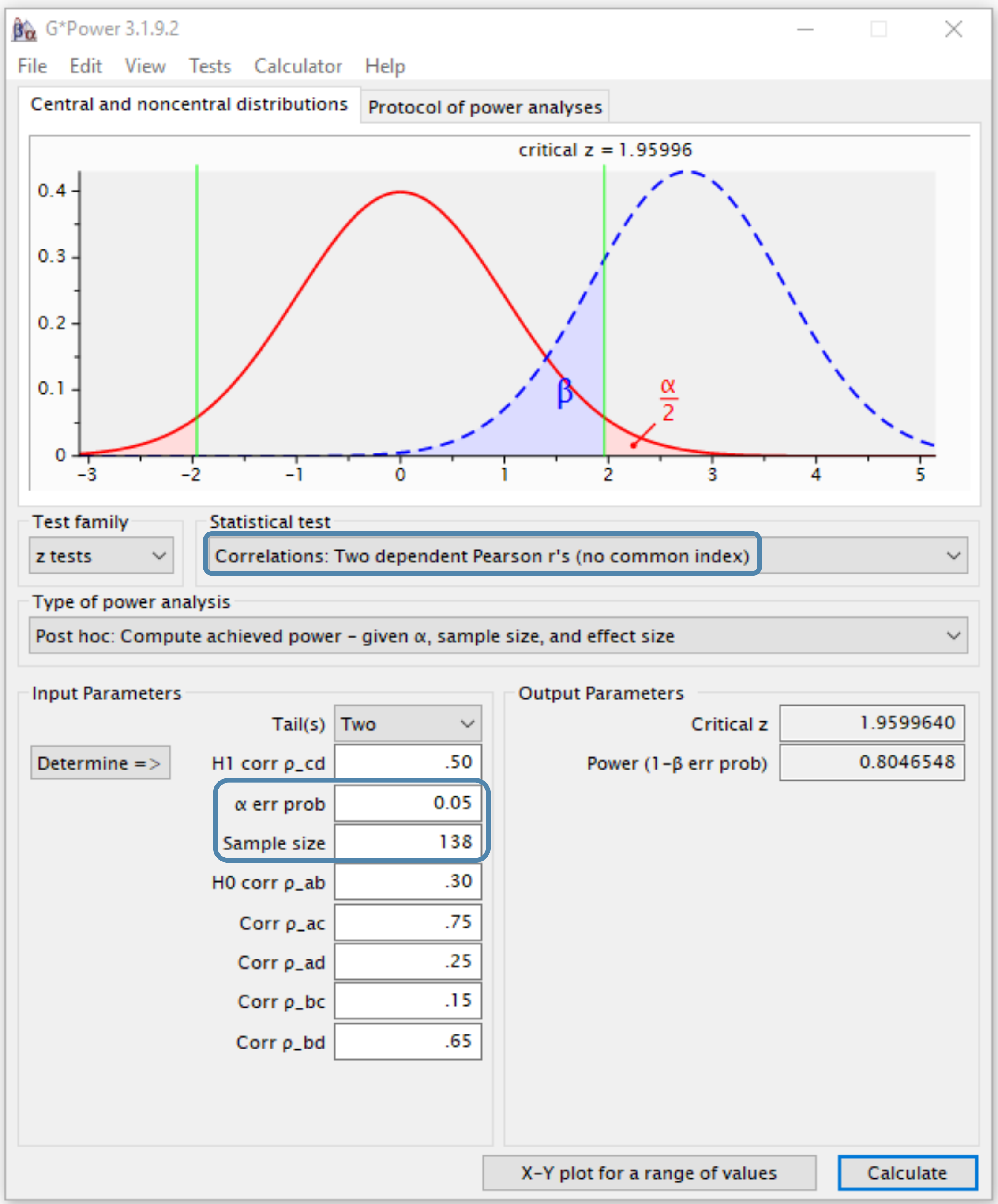




Difference Between Two Non-overlapping Dependent Correlations

			Pre		Post	
			Perf	Mem	Perf	Mem
			a	b	c	d
Pre	Perf	a	1.00	.30	.75	.25
	Mem	b		1.00	.15	.65
Post	Perf	c			1.00	.50
	Mem	d				1.00





Doing Power Analysis with Stata

Comparing Two Means with Clustered Data

Use Stata to Estimate Power for a Cluster Randomized Design

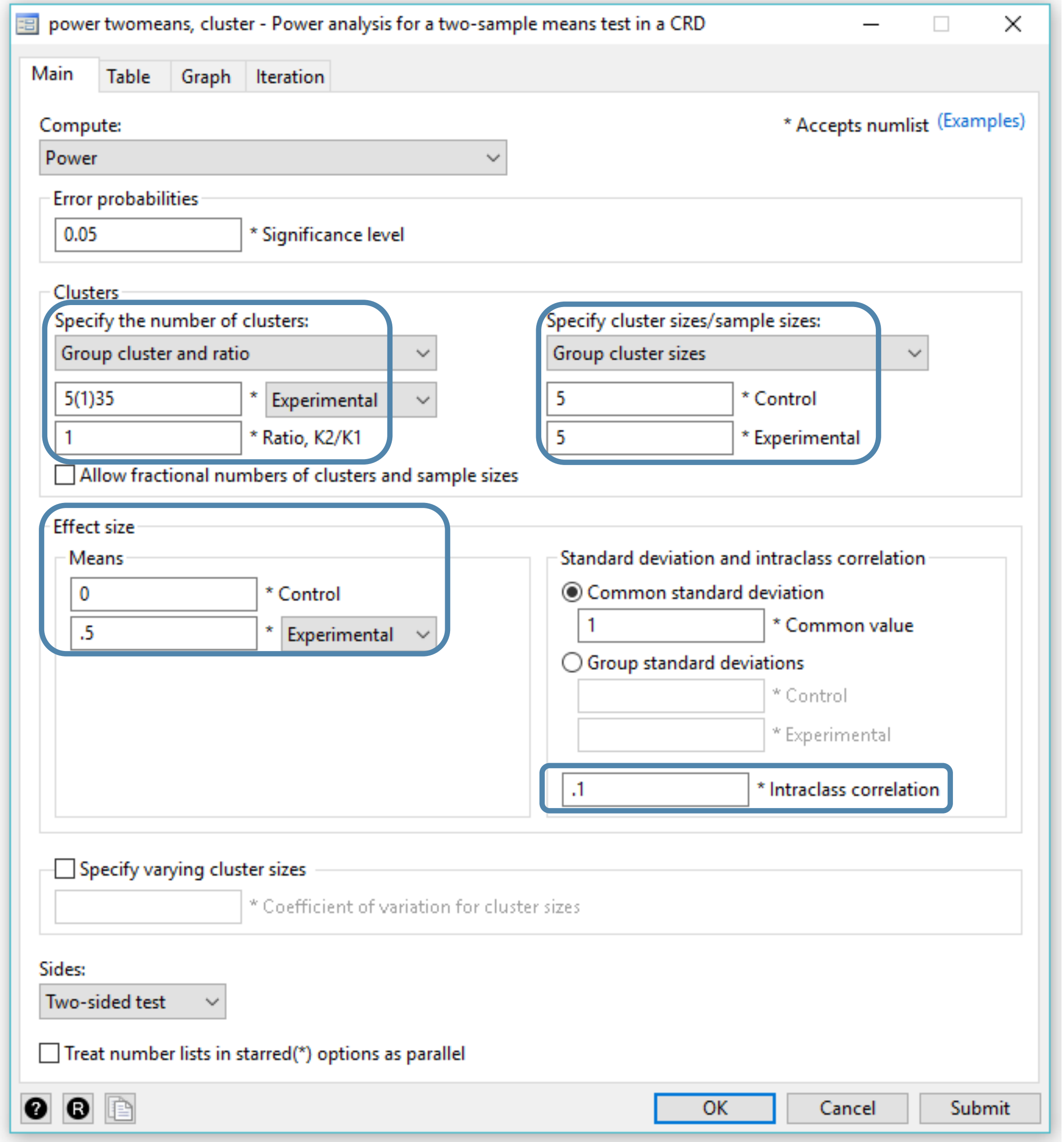
Consider a design with K clusters each having 5 members, in each of two treatments.

Estimate power for designs with K = 5 to 35 clusters

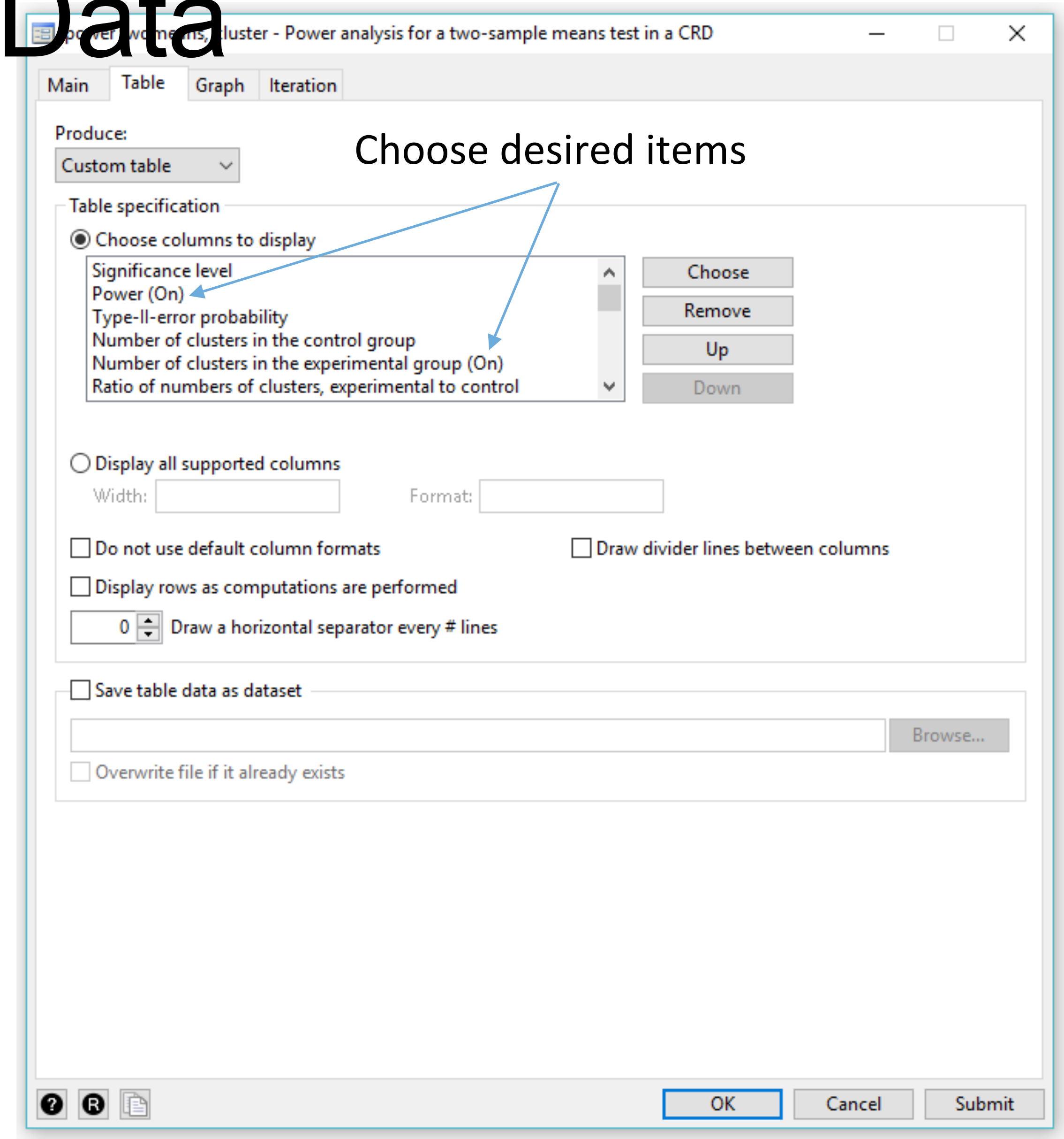
Using a standard (Cohen's d) medium effect size of .5 , with a two-sided significance level of .05, and a .10 Intraclass Correlation

Stata power command

```
power twomeans 0 .5, k2(5(1)35) m1(5) m2(5) rho(.1) ///
 table(power K2 M2 delta rho)                        ///
 graph(ydimension(power) xdimension(K2))
```

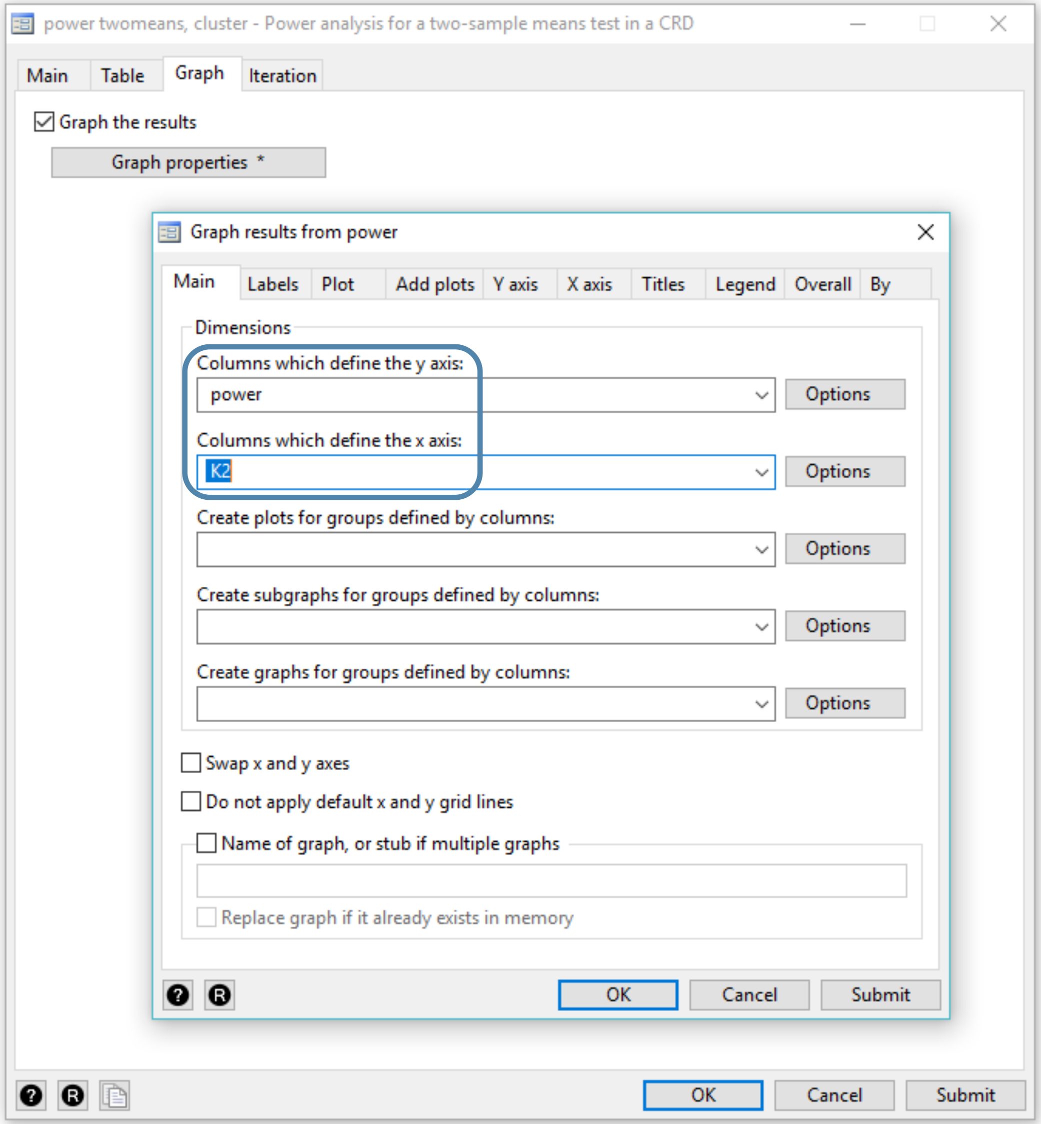


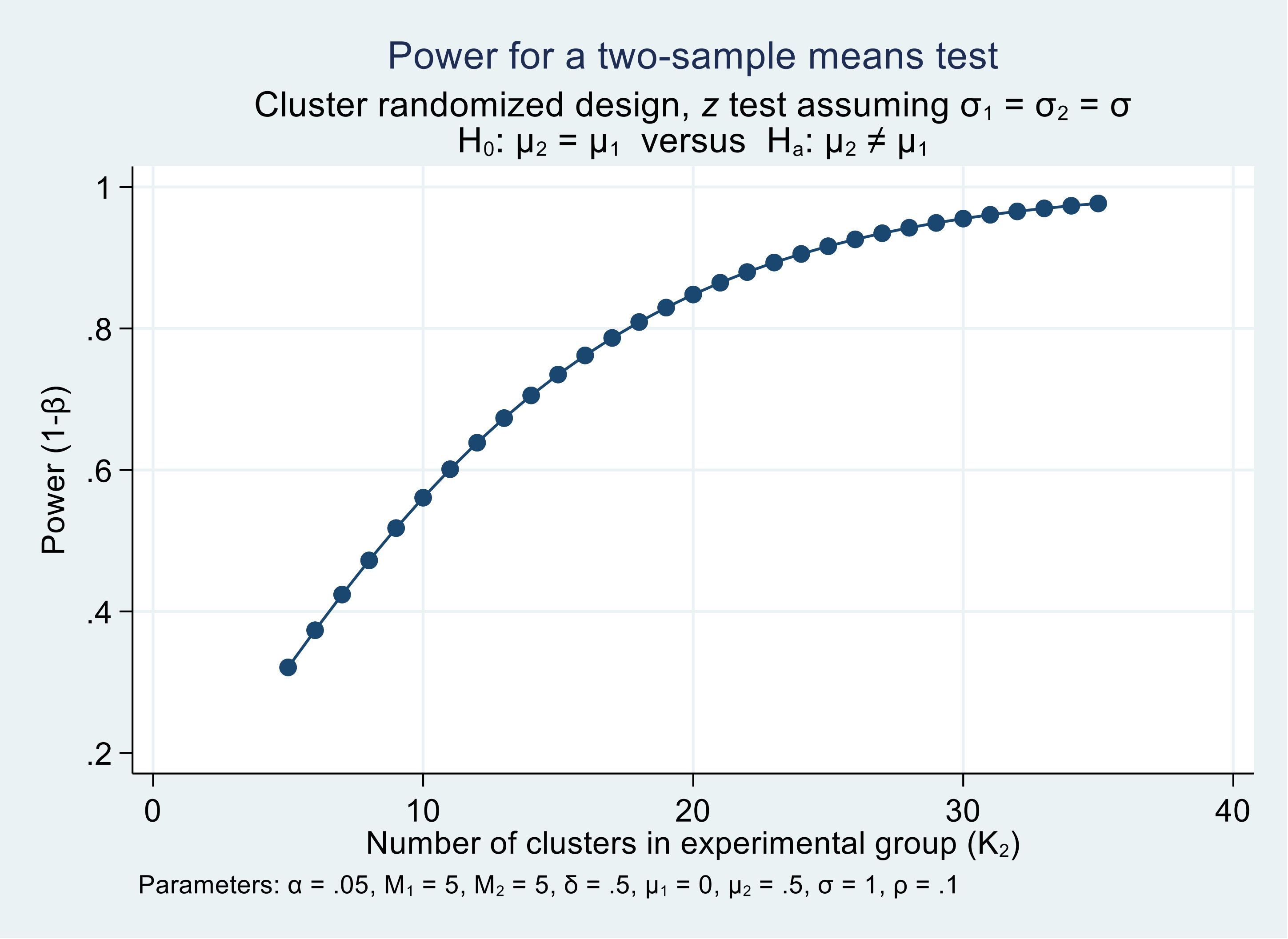
Tabulate Power Results for Clustered Data



power	K2	M2	delta	rho
.3209	5	5	.5	.1
.3734	6	5	.5	.1
.4239	7	5	.5	.1
.4721	8	5	.5	.1
.5178	9	5	.5	.1
.5608	10	5	.5	.1
.6011	11	5	.5	.1
.6386	12	5	.5	.1
.6733	13	5	.5	.1
.7054	14	5	.5	.1
.7349	15	5	.5	.1
.762	16	5	.5	.1
.7867	17	5	.5	.1
.8092	18	5	.5	.1
.8296	19	5	.5	.1
.8481	20	5	.5	.1
.8647	21	5	.5	.1
.8798	22	5	.5	.1
.8933	23	5	.5	.1
.9055	24	5	.5	.1
.9163	25	5	.5	.1
.926	26	5	.5	.1
.9347	27	5	.5	.1
.9424	28	5	.5	.1
.9493	29	5	.5	.1
.9554	30	5	.5	.1
.9608	31	5	.5	.1
.9656	32	5	.5	.1
.9698	33	5	.5	.1
.9736	34	5	.5	.1
.9769	35	5	.5	.1

Plot Power by the Number of Clusters





Estimated Power and Sample Size for the Fit of Structural Equation Models using Selected Web Apps

http://quantpsy.org/rmsea/rmsea.htm

http://timo.gnambs.at/en/scripts/powerforsem

https://webpower.psychstat.org/models/sem01/

- http://quantpsy.org/rmsea/rmsea.htm

Compute Power for RMSEA

Alpha	.05
Degrees of Freedom	20
Sample Size	400
Null RMSEA	0
Alt. RMSEA	.05

Generate R Code

```
#Power analysis for CSM

alpha <- 0.05 #alpha level
d <- 20 #degrees of freedom
n <- 400 #sample size
rmsea0 <- 0 #null hypothesized RMSEA
rmseaa <- 0.05 #alternative hypothesized RMSEA
```

Submit above to Rweb | Erase R code

```
Rweb:> print(pow)
[1] 0.7737188
```

Compute Sample Size for RMSEA

Alpha	.05
Degrees of Freedom	20
Desired Power	.80
Null RMSEA	0
Alt. RMSEA	.05

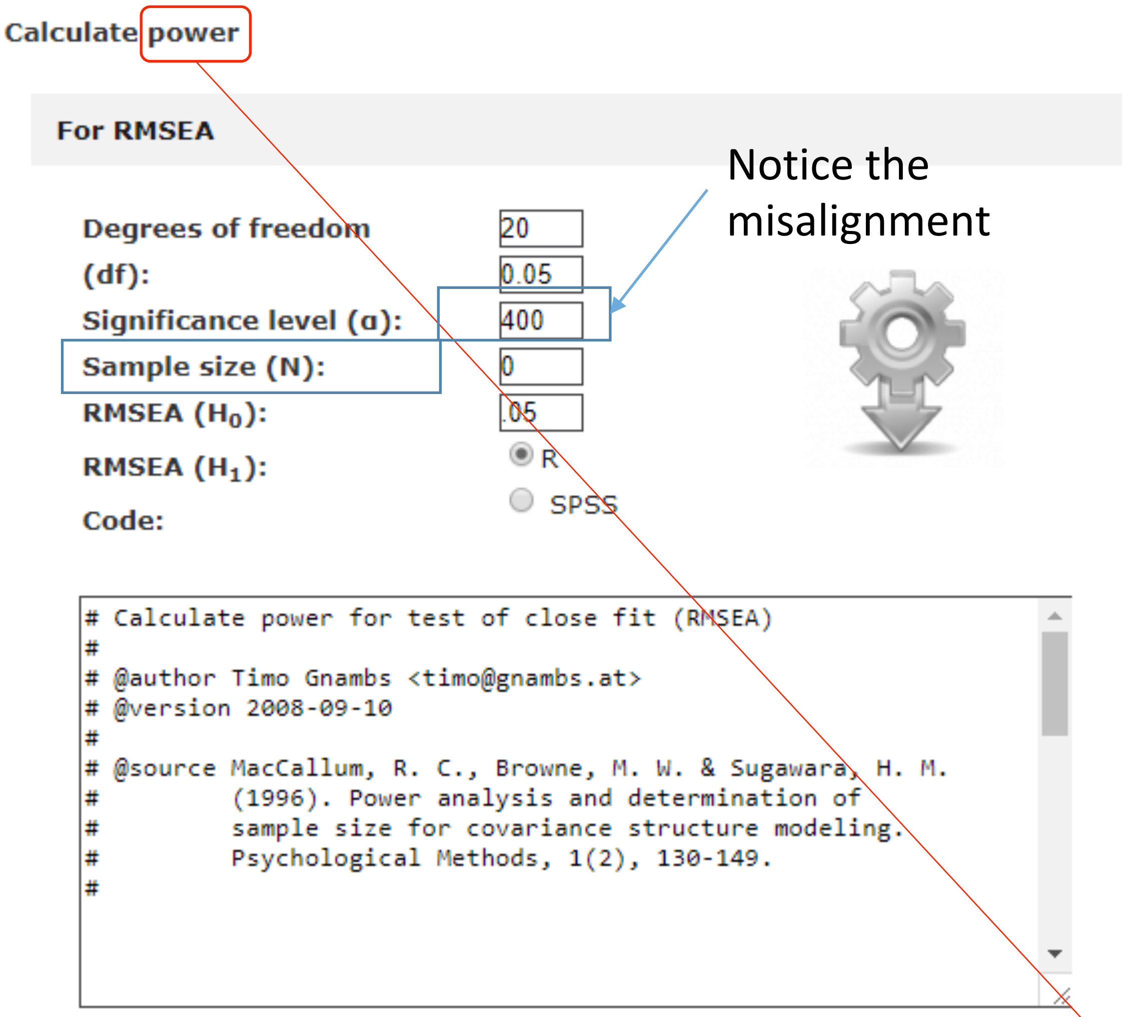
Generate R Code

```
#Computation of minimum sample size for test of fit

rmsea0 <- 0 #null hypothesized RMSEA
rmseaa <- 0.05 #alternative hypothesized RMSEA
d <- 20 #degrees of freedom
alpha <- 0.05 #alpha level
desired <- 0.8 #desired power
```

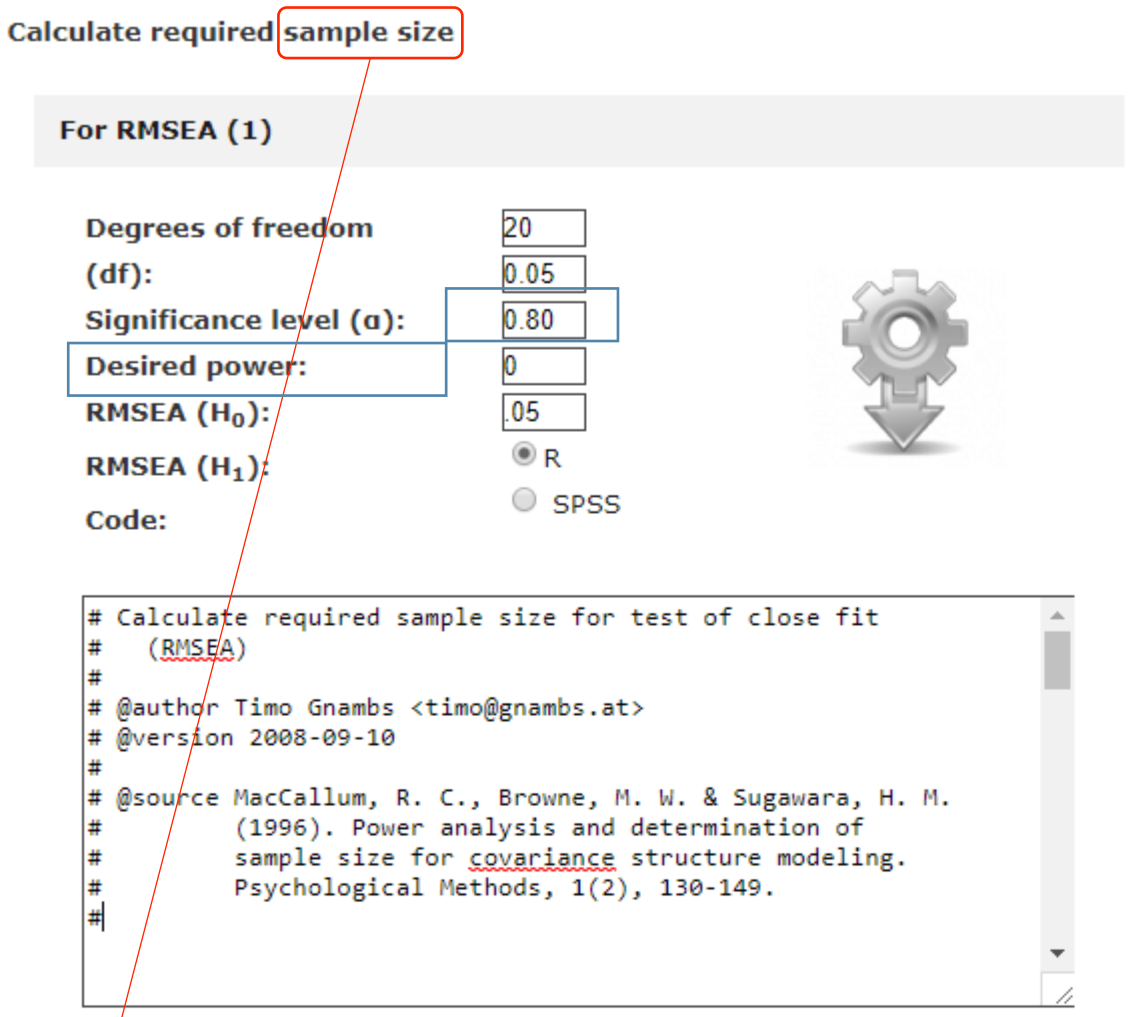
Submit above to Rweb | Erase R code

```
Rweb:> print(minn)
[1] 420.3125
```

- http://timo.gnambs.at/en/scripts/powerforsem



[1] "Power for test of close fit (McCallum et al., 1996)" "0.77"



[1] "Required N for test of close of fit (McCallum et al., 1996)"
[2] "421"

• https://webpower.psychstat.org/models/sem01/

SEM based on RMSEA

Parameters (Help)	
Sample size	400
Degrees of freedom	20
RMSEA for H0	0
RMSEA for H1	.05
Significance level	.05
Power	
Type of analysis	Close fit ▼
Power curve	No power curve ▼
Note	SEM based on RMSEA

Calculate

Output

```
Power for SEM based on RMSEA

      n df rmsea0 rmsea1  power alpha
    400 20      0   0.05 0.7737  0.05

URL: http://psychstat.org/rmsea
```

SEM based on RMSEA

Parameters (Help)	
Sample size	
Degrees of freedom	20
RMSEA for H0	0
RMSEA for H1	.05
Significance level	.05
Power	.8
Type of analysis	Close fit ▼
Power curve	No power curve ▼
Note	SEM based on RMSEA

Calculate

Output

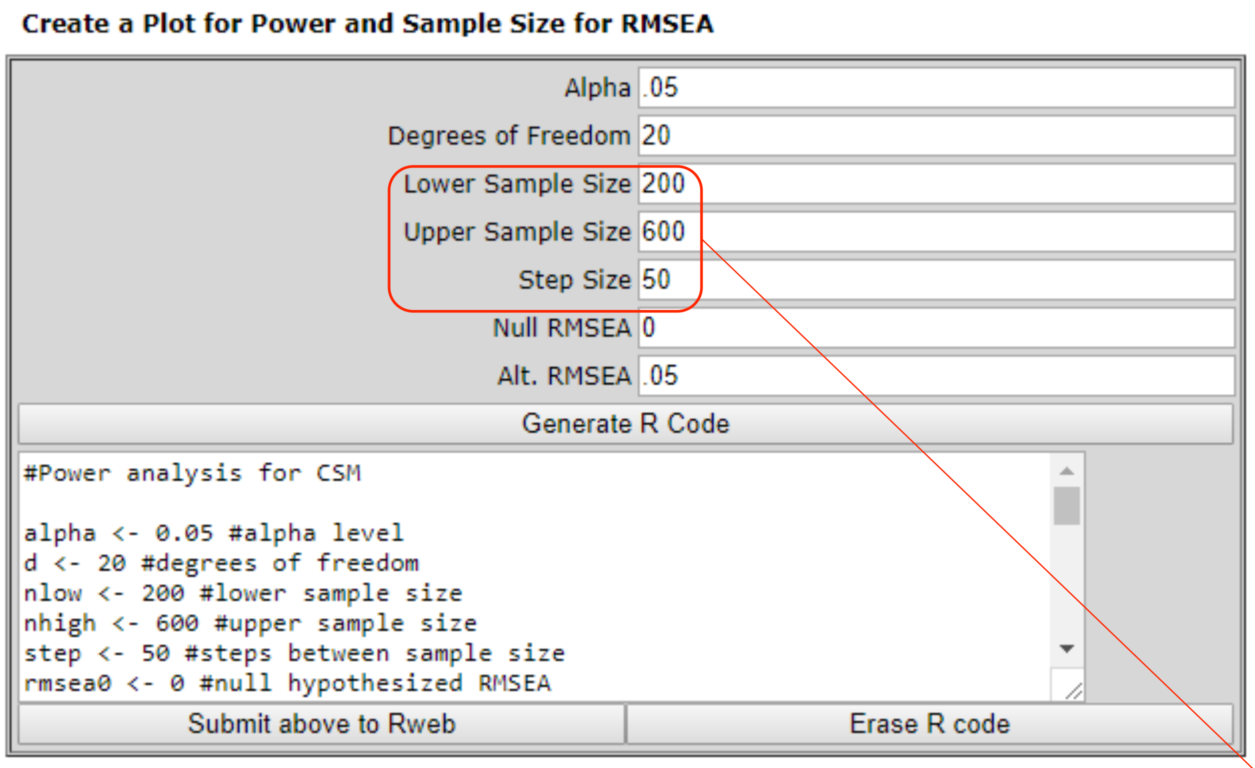
```
Power for SEM based on RMSEA

        n df rmsea0 rmsea1 power alpha
    420.2 20      0   0.05   0.8  0.05

URL: http://psychstat.org/rmsea
```

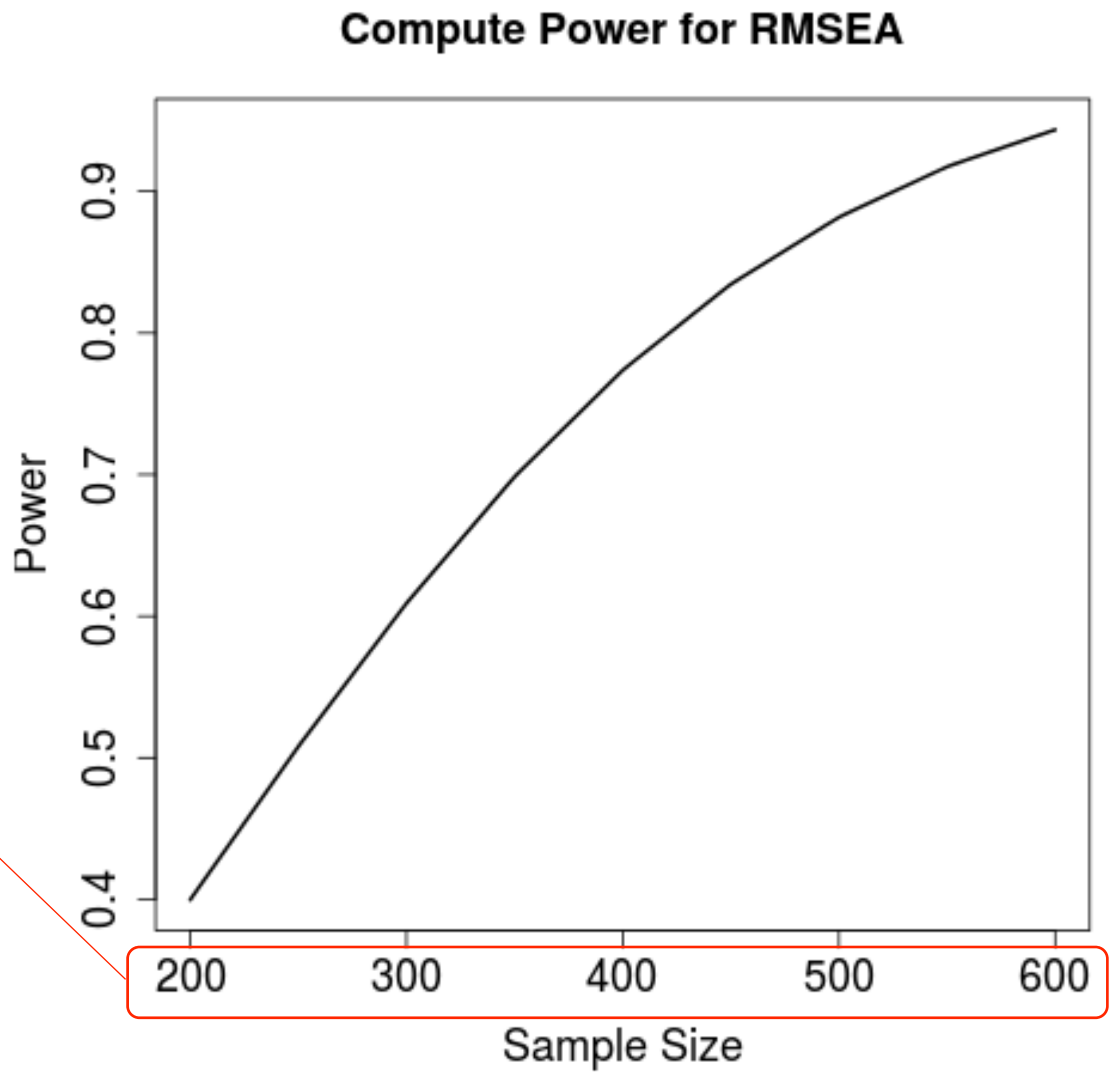
Plotting Power Curves by Sample Size for Structural Equation Models

http://quantpsy.org/rmsea/rmsea.htm

https://webpower.psychstat.org/models/sem01/

- http://quantpsy.org/rmsea/rmseaplot.htm



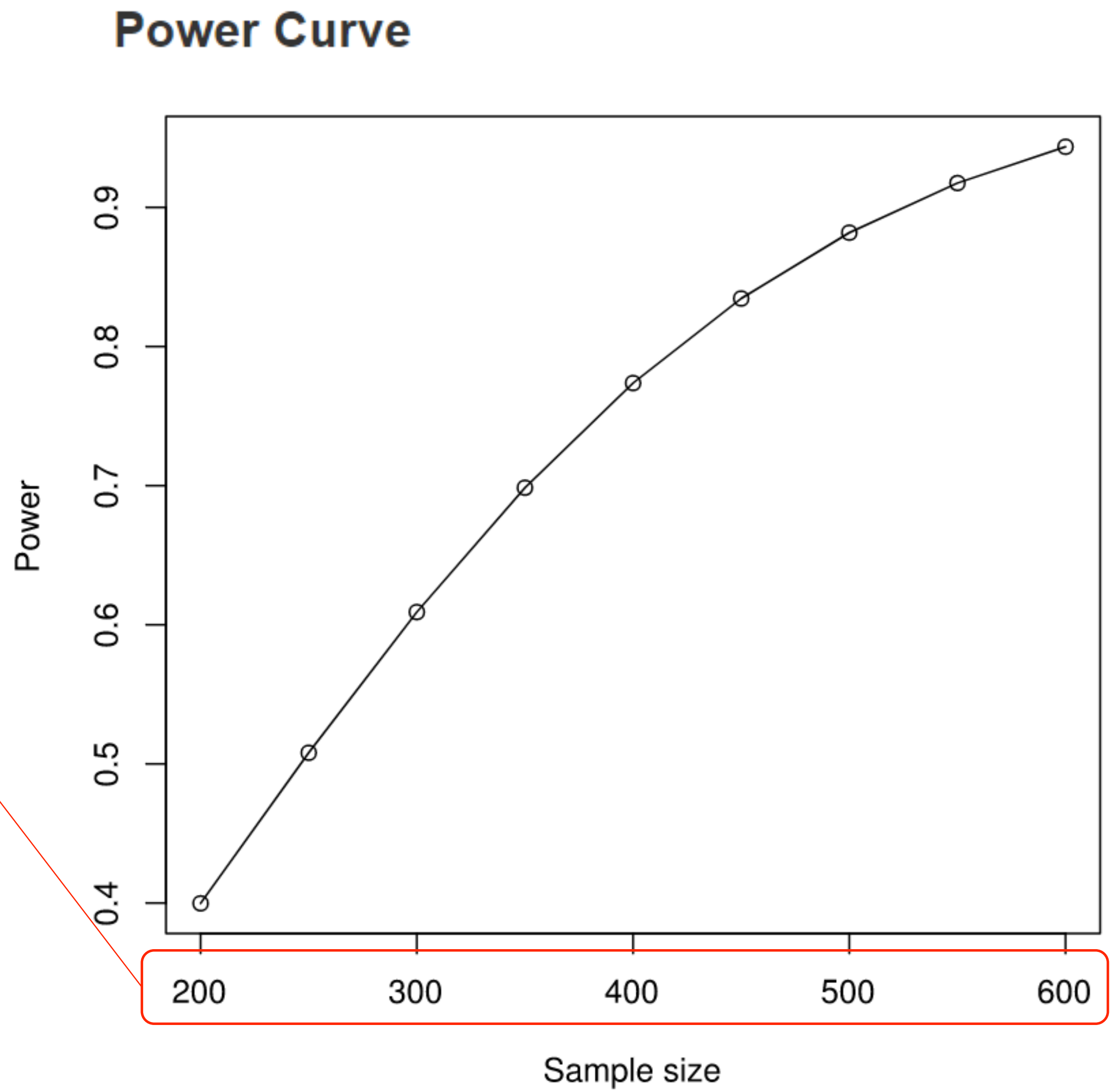


- https://webpower.psychstat.org/models/sem01/

SEM based on RMSEA

Parameters (Help)	
Sample size	200:600:50
Degrees of freedom	20
RMSEA for H0	0
RMSEA for H1	.05
Significance level	.05
Power	
Type of analysis	Close fit ▼
Power curve	Show power curve ▼
Note	SEM based on RMSEA

Calculate



- https://webpower.psychstat.org/models/sem01/

SEM based on RMSEA

Parameters (Help)	
Sample size	200:600:50
Degrees of freedom	20
RMSEA for H0	0
RMSEA for H1	.05
Significance level	.05
Power	
Type of analysis	Close fit ▼
Power curve	Show power curve ▼
Note	SEM based on RMSEA

Calculate

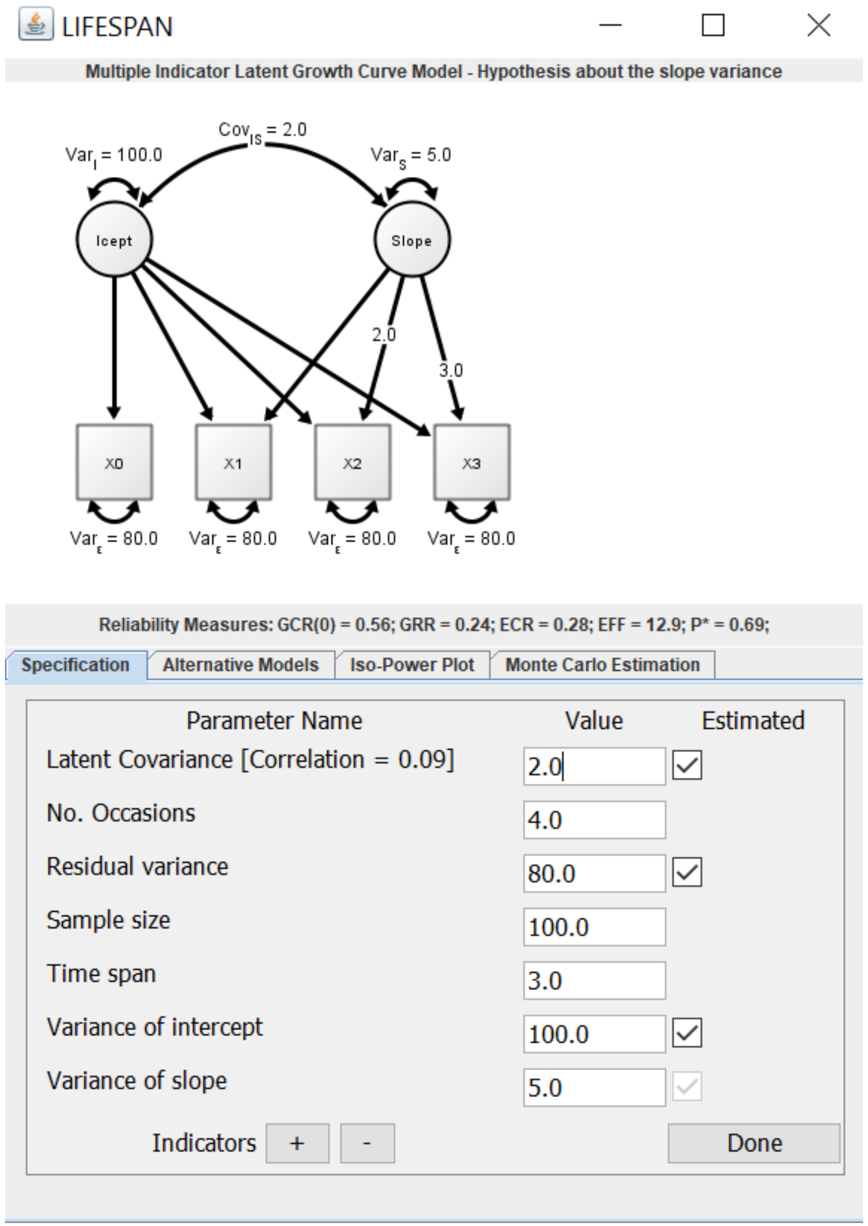
Output

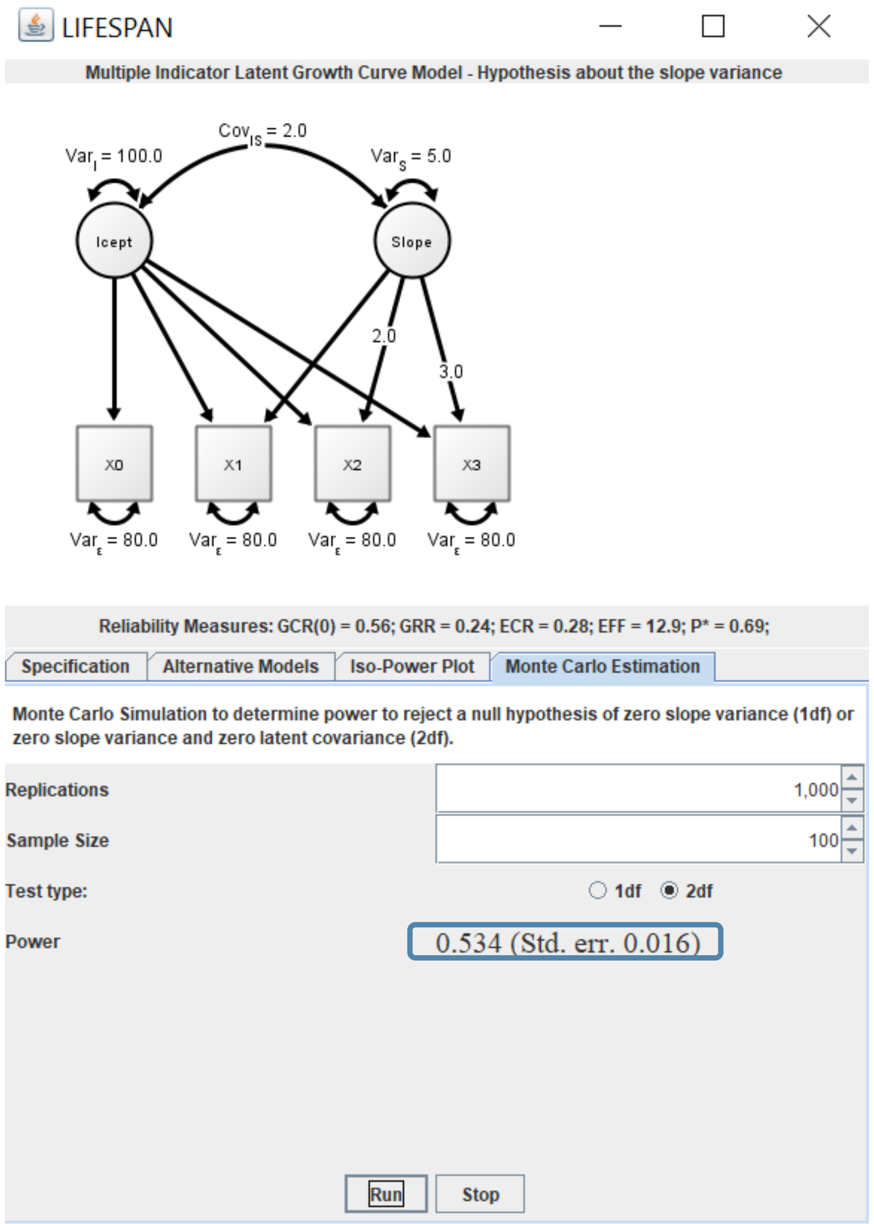
```
Power for SEM based on RMSEA

      n df rmsea0 rmsea1  power alpha
    200 20      0   0.05 0.3997  0.05
    250 20      0   0.05 0.5080  0.05
    300 20      0   0.05 0.6091  0.05
    350 20      0   0.05 0.6985  0.05
    400 20      0   0.05 0.7737  0.05
    450 20      0   0.05 0.8345  0.05
    500 20      0   0.05 0.8818  0.05
    550 20      0   0.05 0.9175  0.05
    600 20      0   0.05 0.9436  0.05

URL: http://psychstat.org/rmsea
```

Power for Latent Growth Curve Models

http://www.brandmaier.de/lifespan/

- http://www.brandmaier.de/lifespan/





Power and Sample Size for the Actor-Partner Interdependence Model (APIM)

https://robert-a-ackerman.shinyapps.io/APIMPowerRdis/

https://robert-a-ackerman.shinyapps.io/APIMPowerRdis/

Power Analysis for the Actor-Partner Interdependence Model

Task | Effect Size | Miscellaneous

Study Information: Begin Computations

Compute:

- (•) Power Given N
- ( ) N Given Desired Power

Power or sample size

Dyad Type:

- ( ) Indistinguishable
- (•) Distinguishable

Distinguishable or indistinguishable

To use labels other than 'Person 1' and 'Person 2' for the inputs, please modify the labels in the boxes below.

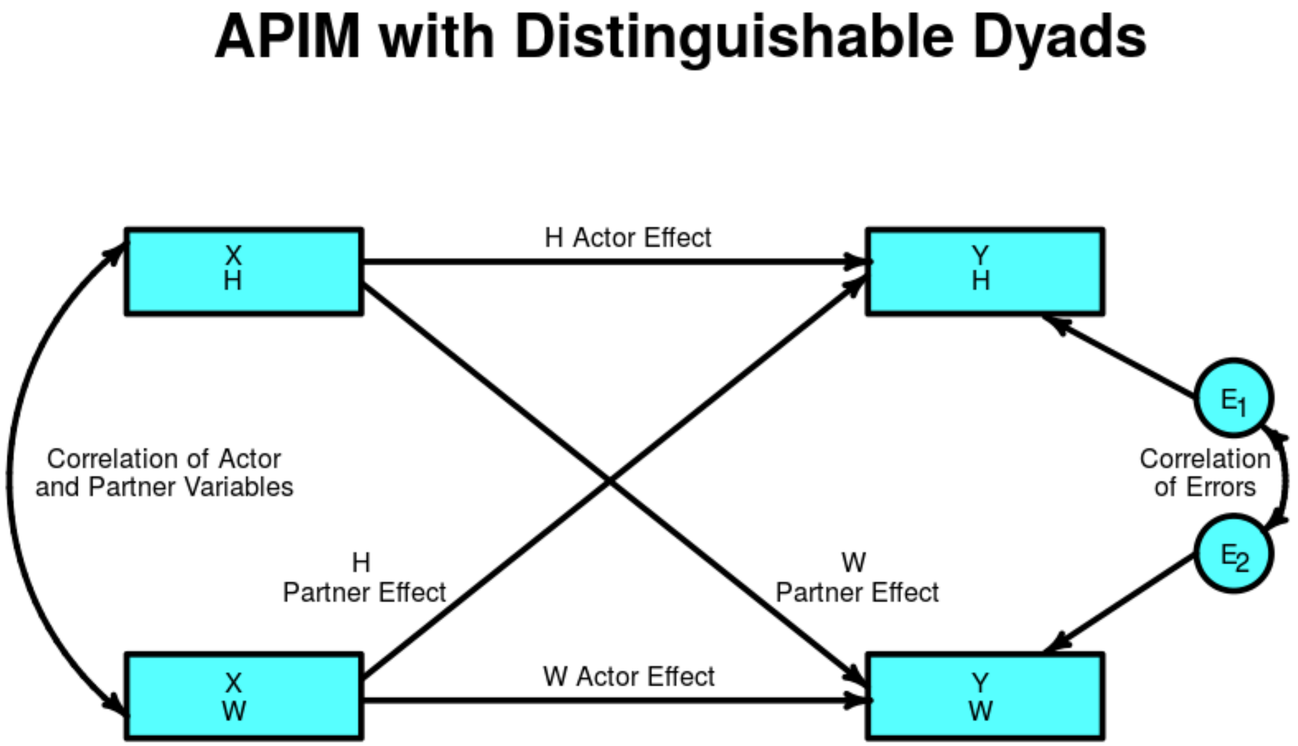
Label for Person 1:

H

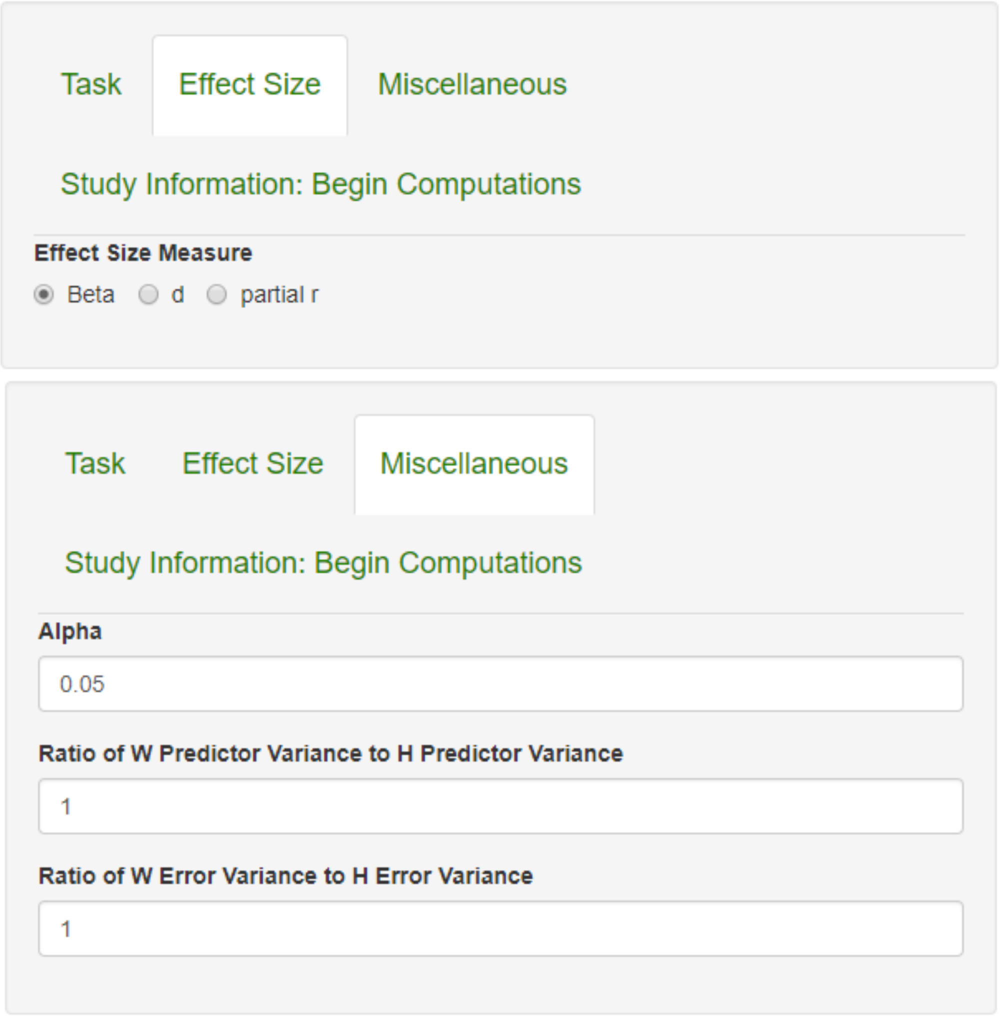
Label for Person 2:

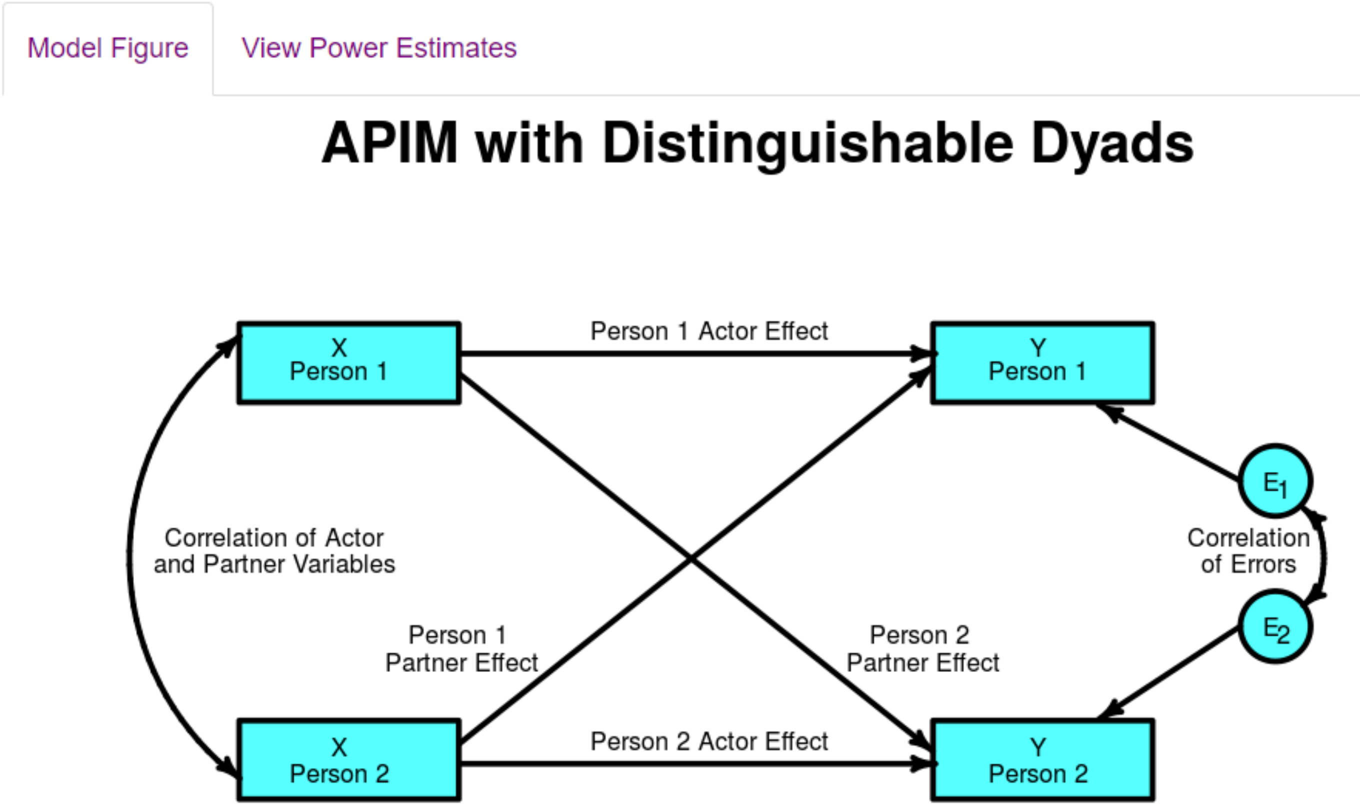
W

Model Figure | View Power Estimates



https://robert-a-ackerman.shinyapps.io/APIMPowerRdis/





Task Effect Size Miscellaneous

Study Information: Begin Computations

Click the 'Compute Power!' button below when you are ready to conduct the analysis.

Compute Power!

Effect Size Value of the Actor Effect for H

0.25

Effect Size Value of the Actor Effect for W

0.25

Effect Size Value of the Partner Effect for H

0.15

Effect Size Value of the Partner Effect for W

0.15

Correlation of the Actor and Partner Variables

0.3

Correlation of the Errors

0.3

Number of Dyads with Complete Data (at least 10)

100

Number of H Singles (if non-zero, at least 5)

0

Number of W Singles (if non-zero, at least 5)

0

Model Figure View Power Estimates

	Effect	Power	N	df	Beta	r	partial r	ncp
Actor Effect for H	.250	.692	100	97	.250	.295	.245	2.486
Actor Effect for W	.250	.692	100	97	.250	.295	.245	2.486
Partner Effect for H	.150	.315	100	97	.150	.225	.150	1.492
Partner Effect for W	.150	.315	100	97	.150	.225	.150	1.492
Difference in Actor Effects	.000	.050						0.000
Difference in Partner Effects	.000	.050						0.000
Average of Actor Effects	.250	.920						3.368
Average of Partner Effects	.150	.524						2.021

The task is to determine the levels of power available to detect the actor and partner effects for an Actor-Partner Interd
sample size and alpha.

Alpha is set to .050. N refers to the number of dyads. There are 100 dyads (and 0 H singles and 0 W singles).

The measure of effect size is beta, the standardized regression coefficient. The correlation between the two members'
the non-centrality parameter or the regression coefficient divided by its standard error.

Truncated on the slide

There is .692 power to detect an actor effect for H of size .250 (i.e., a beta of .250 or a partial r of .245).

There is .692 power to detect an actor effect for W of size .250 (i.e., a beta of .250 or a partial r of .245).

There is .315 power to detect a partner effect for H of size .150 (i.e., a beta of .150 or a partial r of .150).

There is .315 power to detect a partner effect for W of size .150 (i.e., a beta of .150 or a partial r of .150).

There is .050 power to detect a difference between the H and W actor effects of size .000.

There is .050 power to detect a difference between the H and W partner effects of size .000.

There is .920 power to detect average of the H and W actor effects of size .250.

There is .524 power to detect average of the H and W partner effects of size .150.

Power and Sample Size for Simple Mediation Analysis

https://davidakenny.shinyapps.io/MedPower

https://schoemanna.shinyapps.io/mc_power_med

https://webpower.psychstat.org/models/diagram

- https://davidakenny.shinyapps.io/MedPower

Power and N Computations for Mediation

Compute Now!

Determine:

- (●) Power given Sample Size
- ( ) Sample size given desired level of power

Sample Size

100

Effect Size Measure

(●) Beta ( ) partial r

Effect of X on M (path a)

.400

Effect of M on Y (path b)

.381

Effect of X on Y (path c')

.048

Alpha

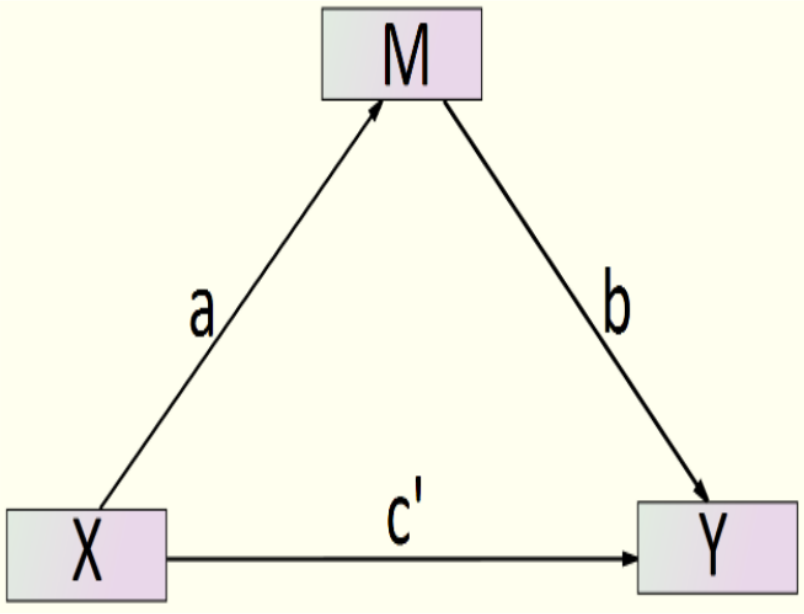
.05

Please click here to make a small donation of $2.50 to offset some of the costs of maintaining MedPower.

Effect	Beta	Partial r	Power	N
c (total)	.200	.200	.518	100
a	.400	.400	.990	100
b	.381	.356	.961	100
c' (direct)	.048	.048	.076	100
ab (indirect)	.152		.951	100

Alpha for all power calculations set to .050. Effects (a, b, and c') are Betas.

Mediation Diagram



- https://schoemanna.shinyapps.io/mc_power_med

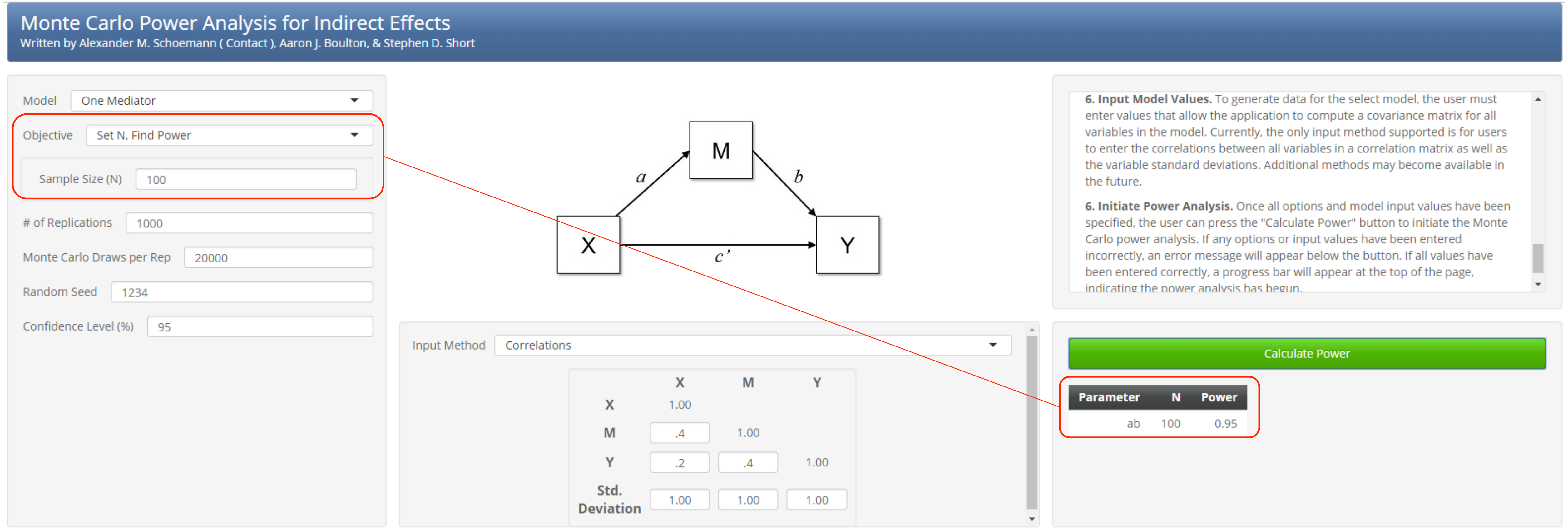
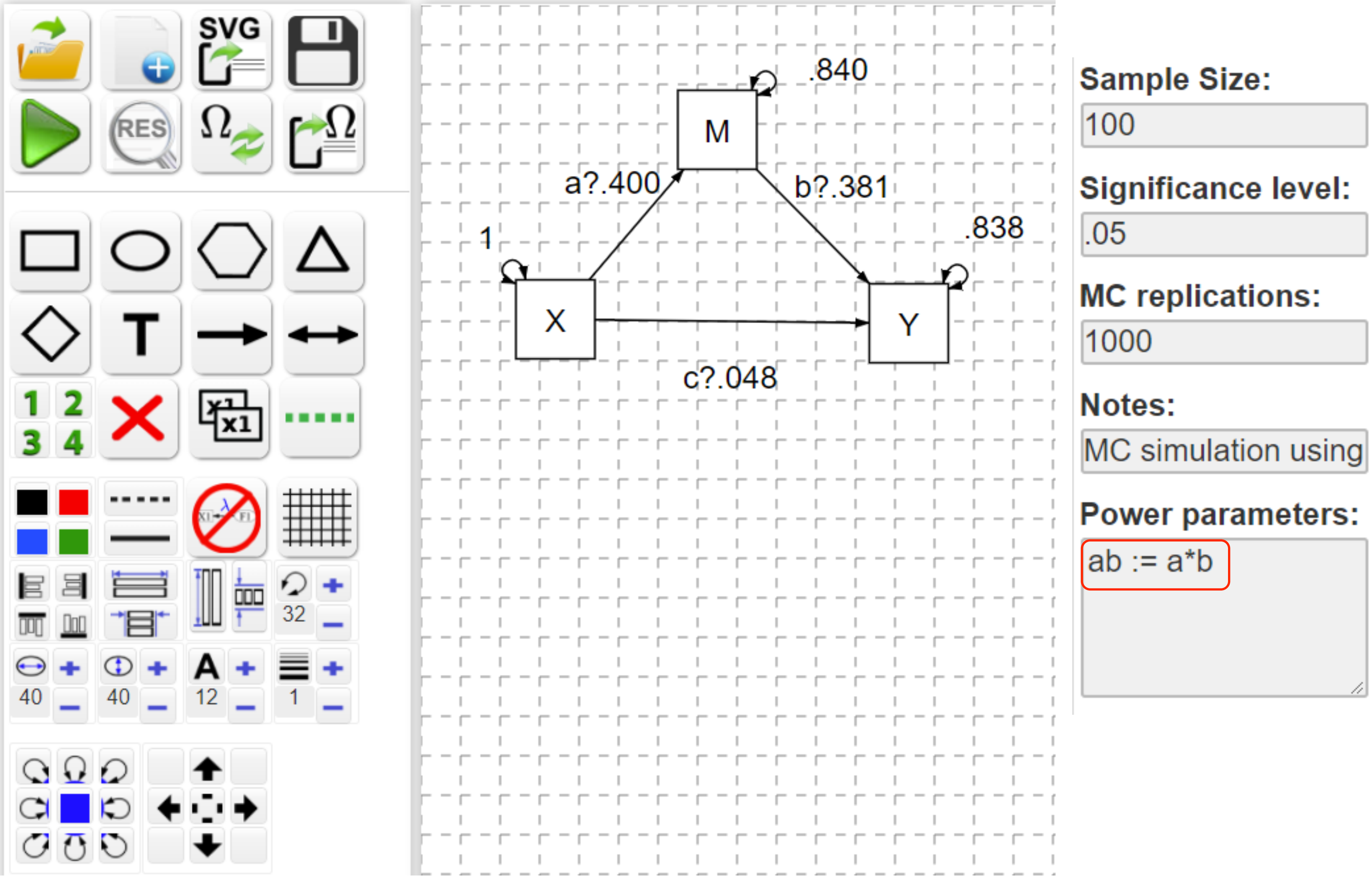


Diagram based Power analysis through Monte Carlo simulation (Diagram input)

- https://webpower.psychstat.org/models/diagram/



"Use with caution because the application … needs more testing."

Diagram based Power analysis through Monte Carlo simulation (Text Output)

```
Basic information:

  Esimation method                                    ML
  Standard error                                standard
  Number of requested replications                  1000
  Number of successful replications                 1000
  Sample size                                        100

                       True  Estimate      MSE       SD     Power  Power.se Coverage
Regressions:
  M ~
    X          (a)     0.400    0.398    0.092    0.095    0.991    0.003    0.933
  Y ~
    M          (b)     0.381    0.378    0.100    0.101    0.959    0.006    0.940
    X          (c)     0.048    0.049    0.100    0.108    0.096    0.009    0.934

Intercepts:
    M                  0.000   -0.002    0.091    0.090    0.043    0.006    0.957
    Y                  0.000   -0.002    0.091    0.091    0.049    0.007    0.951
    X                  0.000    0.000    0.099    0.100    0.049    0.007    0.951

Variances:
    M                  0.840    0.828    0.117    0.116    1.000    0.000    0.925
    Y                  0.838    0.815    0.115    0.119    1.000    0.000    0.916
    X                  1.000    0.987    0.140    0.140    1.000    0.000    0.938

Indirect/Mediation effects:
    ab                 0.152    0.151    0.054    0.054    0.900    0.009    0.925
```

Using Simulation in Mplus to Estimate Power (Simple Mediation Analysis)

Simulation for Power Estimation—Step 1: Estimate Standardized Model Parameters from Correlations

```
TITLE: Step 1: Estimate standardized mediation
 parameters from correlation matrix input
DATA: FILE IS medcorr.dat;   ! Data file
 NOBSERVATIONS = 100;        ! Sample size
 TYPE=CORRELATION;           ! Correlation matrix input
VARIABLE: NAMES ARE x m y;   ! Variable names
ANALYSIS: ESTIMATOR IS GLS;  ! Generalized least squares
MODEL: x@1;                  ! Fix variance to 1
 m ON x;                     ! First-stage effect
 y ON m;                     ! Second-stage effect
 y ON x;                     ! Direct effect
MODEL INDIRECT:              ! Mediation parameters
 y IND x;                    ! Indirect effect
OUTPUT: SAMPSTAT;            ! Descriptive statistics
 STDYX;                      ! Standardized results
```

```
                    Estimate       S.E.  Est./S.E.    P-Value
 M        ON
    X                  0.400      0.092      4.342      0.000
 Y        ON
    M                  0.381      0.100      3.794      0.000
    X                  0.048      0.100      0.474      0.635
 Variances
    X                  1.000      0.000    999.000    999.000
 Residual Variances
    M                  0.840      0.119      7.035      0.000
    Y                  0.838      0.119      7.036      0.000
...
  Total                0.200      0.098      2.031      0.042
  Indirect             0.152      0.053      2.857      0.004
  Direct               0.048      0.100      0.474      0.635
```

medcorr.dat

```
1
.4 1
.2 .4 1
```

Simulation for Power Estimation—Step 2: Code to Estimate Power for Mediation Parameters

	Estimate	S.E.	Est./S.E.	P-Value
M ON				
X	0.400	0.092	4.342	0.000
Y ON				
M	0.381	0.100	3.794	0.000
X	0.048	0.100	0.474	0.635
Variances				
X	1.000	0.000	999.000	999.000
Residual Variances				
M	0.840	0.119	7.035	0.000
Y	0.838	0.119	7.036	0.000
...				
Total	0.200	0.098	2.031	0.042
Indirect	0.152	0.053	2.857	0.004
Direct	0.048	0.100	0.474	0.635

```
TITLE: Step 2: Simulate mediation parameters
 to obtain estimated power
MONTECARLO:                        ! Monte Carlo Simulation
 NAMES ARE x m y;                  ! Variable names
 NOBSERVATIONS ARE 100;            ! Sample size
 SEED = 20160129;                  ! Seed
 NREPS=1000;                       ! Number of replications
MODEL POPULATION:                  ! Population model
 y ON x*.048;                      ! Direct effect
 y ON m*.381;                      ! Second stage effect
 m ON x*.400;                      ! First stage effect
 x@1 m*.840 y*.838;                ! Variances
ANALYSIS: ESTIMATOR IS GLS;        ! Generalized least squares
 MODEL IS NOMEANSTRUCTURE;         ! Omit mean structure
MODEL:                             ! Analysis model
 y ON x*.048;                      ! Direct effect
 y ON m*.381;                      ! Second stage effect
 m ON x*.400;                      ! First stage effect
 x@1 m*.840 y*.838;                ! Variances
MODEL INDIRECT:                    ! Mediation analysis
 y IND x;                          ! Indirect effect
```

Simulation for Power Estimation—Step 2: Mediation Parameter Power Estimates

	Population	ESTIMATES Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
X	0.048	0.0497	0.0991	0.0995	0.0098	0.954	0.073
M	0.381	0.3812	0.1032	0.1001	0.0106	0.938	0.967
M ON							
X	0.400	0.3999	0.0913	0.0914	0.0083	0.940	0.992
Variances							
X	1.000	1.0000	0.0000	0.0000	0.0000	1.000	0.000
Residual Variances							
M	0.840	0.8357	0.1186	0.1188	0.0141	0.940	1.000
Y	0.838	0.8218	0.1247	0.1168	0.0158	0.909	1.000
...							
Total	0.200	0.2022	0.0983	0.0976	0.0097	0.935	0.542
Indirect	0.152	0.1525	0.0549	0.0541	0.0030	0.934	0.914
Direct	0.048	0.0497	0.0991	0.0995	0.0098	0.954	0.073

Power

Second Stage

First Stage

Total

Indirect

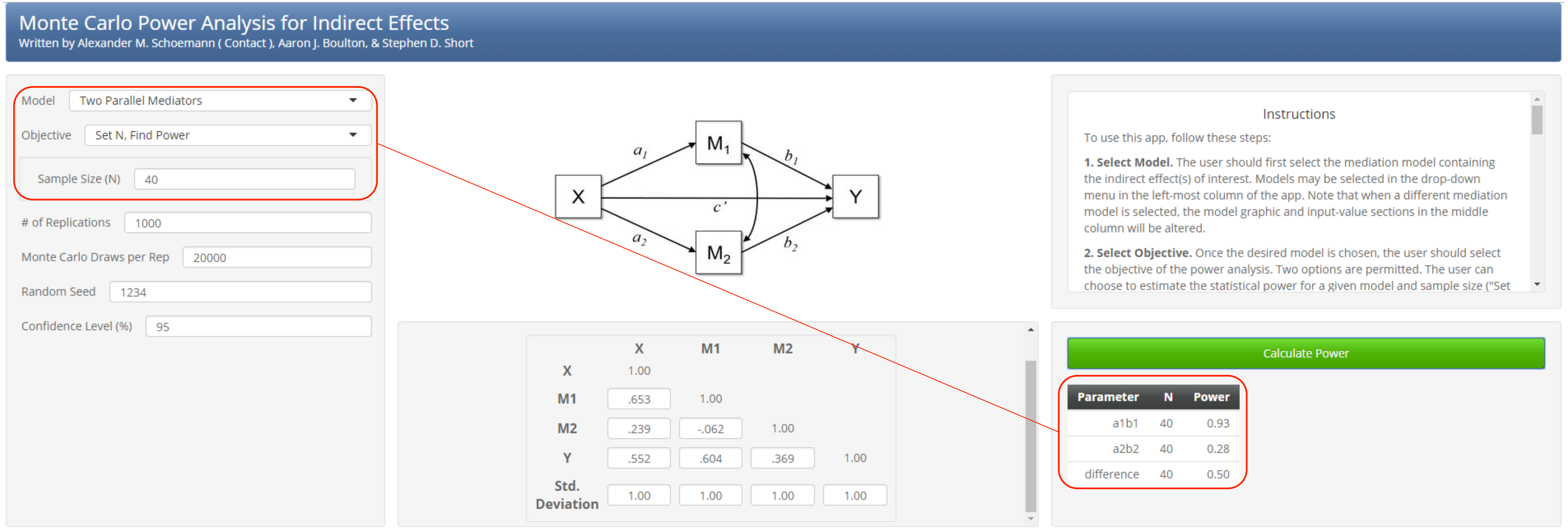
Direct

Power and Sample Size Calculation for Extended Mediation Analysis

https://schoemanna.shinyapps.io/mc_power_med

- https://schoemanna.shinyapps.io/mc_power_med

Two Parallel Mediators: Standardized Effects



Simulation for Power Estimation—Step 1: Estimate Standardized Population Mediation Parameters

```
TITLE: Step 1: Find population mediation parameters
DATA: FILE IS "twomedcorr.dat"; ! File name
 TYPE IS CORRELATION ;          ! Correlations
 NOBSERVATIONS ARE 40;          ! Sample size
VARIABLE: NAMES ARE x m1 m2 y;  ! Variable names
ANALYSIS: ESTIMATOR IS GLS;     ! GLS estimation
MODEL:                          ! Mediation model
 y ON m1 m2 (b1 b2);            ! Second stage effects
 m1 m2 ON x (a1 a2);            ! First stage effects
 y ON x;                        ! Direct effect
 m1 WITH m2;                    ! Mediator covariance
MODEL INDIRECT: y IND x;        ! Indirect effects
MODEL CONSTRAINT:               ! Model constraint
 NEW(diff); diff=a1*b1-a2*b2;   ! Difference parameter
OUTPUT: STDYX SAMPSTAT;         ! Output
```

```
                       Estimate      S.E.  Est./S.E.    P-Value
 Y        ON
    M1                    0.570     0.151      3.776      0.000
    M2                    0.383     0.118      3.254      0.001
    X                     0.088     0.155      0.567      0.571
 M1       ON
    X                     0.653     0.121      5.384      0.000
 M2       ON
    X                     0.239     0.156      1.537      0.124
 M1       WITH
    M2                   -0.218     0.123     -1.775      0.076
 Variances
    X                     1.000     0.226      4.415      0.000
 Residual Variances
    M1                    0.574     0.130      4.416      0.000
    M2                    0.943     0.214      4.416      0.000
    Y                     0.466     0.105      4.416      0.000
New/Additional Parameters
    DIFF                  0.281     0.140      2.003      0.045
```

twomedcorr.dat

1.000			
0.653	1.000		
0.239	-0.062	1.000	
0.552	0.604	0.369	1.000

Step 2: Simulate to Estimate Power for Model Parameters

	Estimate	S.E.	Est./S.E.	P-Value
Y ON				
M1	0.570	0.151	3.776	0.000
M2	0.383	0.118	3.254	0.001
X	0.088	0.155	0.567	0.571
M1 ON				
X	0.653	0.121	5.384	0.000
M2 ON				
X	0.239	0.156	1.537	0.124
M1 WITH				
M2	-0.218	0.123	-1.775	0.076
Variances				
X	1.000	0.226	4.415	0.000
Residual Variances				
M1	0.574	0.130	4.416	0.000
M2	0.943	0.214	4.416	0.000
Y	0.466	0.105	4.416	0.000
New/Additional Parameters				
DIFF	0.281	0.140	2.003	0.045

```
TITLE: Step 2: Simulate mediation parameters
 to obtain estimated power
MONTECARLO:                             ! Monte Carlo Simulation
 NAMES ARE x m1 m2 y;                   ! Variable names
 NOBSERVATIONS ARE 40;                  ! Sample size
 SEED = 20160129;                       ! Seed
 NREPS=1000;                            ! Number of replications
MODEL POPULATION:                       ! Population model
 y ON m1*.570;                          ! Second stage mediator 1
 y ON m2*.383;                          ! Second stage mediator 2
 m1 ON x*.653;                          ! First stage mediator 1
 m2 ON x*.239;                          ! First stage mediator 2
 y ON x*.088;                           ! Direct effect
 m1 WITH m2*-.218;                      ! Mediator covariance
 x@1 m1*.574 m2*.943 y*.466;            ! Variances
ANALYSIS: ESTIMATOR IS GLS;             ! GLS estimation
 MODEL IS NOMEANSTRUCTURE;              ! No means or intercepts
MODEL:                                  ! Analysis model
 y ON m1*.570 (b1);                     ! Second stage mediator 1
 y ON m2*.383 (b2);                     ! Second stage mediator 2
 m1 ON x*.653 (a1);                     ! First stage mediator 1
 m2 ON x*.239 (a2);                     ! First stage mediator 2
 y ON x*.088;                           ! Direct effect
 m1 WITH m2*-.218;                      ! Mediator covariance
 x@1 m1*.574 m2*.943 y*.466;            ! Variances
MODEL INDIRECT:                         ! Mediation analysis
 y IND x;                               ! Indirect effect
MODEL CONSTRAINT:                       ! Model constraint
 NEW(diff*.281);                        ! Difference
 diff=(a1*b1)-(a2*b2);                  !
```

Mplus Output Estimating Power for Model Parameters

	ESTIMATES Population	Average	Std. Dev.	S. E. Average	M. S. E.	95% Cover	% Sig Coeff
Y ON							
M1	0.570	0.5709	0.1530	0.1501	0.0234	0.935	0.960
M2	0.383	0.3860	0.1274	0.1183	0.0162	0.931	0.870
X	0.088	0.0876	0.1591	0.1526	0.0253	0.933	0.102
M1 ON							
X	0.653	0.6470	0.1258	0.1186	0.0159	0.918	0.999
M2 ON							
X	0.239	0.2477	0.1583	0.1508	0.0251	0.910	0.371
M1 WITH							
M2	-0.218	-0.2095	0.1211	0.1193	0.0147	0.935	0.419
Variances							
X	1.000	1.0000	0.0000	0.0000	0.0000	1.000	0.000
Residual Variances							
M1	0.574	0.5637	0.1335	0.1276	0.0179	0.910	1.000
M2	0.943	0.9121	0.2202	0.2065	0.0494	0.880	1.000
Y	0.466	0.4290	0.1026	0.0972	0.0119	0.859	1.000
New/Additional Parameters							
DIFF	0.281	0.2722	0.1457	0.1426	0.0213	0.934	0.479
...							
Effects from X to Y							
Total	0.552	0.5533	0.1375	0.1302	0.0189	0.911	0.989
Tot indirect	0.464	0.4657	0.1390	0.1371	0.0193	0.939	0.972
Specific indirect							
M1	0.372	0.3690	0.1226	0.1206	0.0150	0.930	0.942
M2	0.092	0.0968	0.0724	0.0684	0.0053	0.912	0.199

Power

Second stage effects

Direct effect

First stage effects

Difference between the indirect effects via the 2 mediators

Total effect

Indirect effects

Summarizing

- Background and statistical foundations of power analysis
- Software tools for power analysis
    - General purpose commercial statistical software (Stata, SAS, SPSS)
    - Freely available software (G*Power, PS)
    - Web Apps
- Formula-based vs. general simulation approaches to power analysis

Related issues and methods

- Confidence Intervals (CI) and Accuracy in Parameter Estimation (AIPE)
- Effect size (ES) estimation and conversion
- Power for categorical, count, or censored variables
- Power for ANOVA models
- Power for tests of interaction and moderation
- Power for complex multilevel models
- Power for equivalence and non-inferiority testing
- Power and meta-analysis
- Power analysis and sensitivity analysis

Learning More about Statistical Power Analysis

- Aberson, C. L. (2011). *Applied Power Analysis for the Behavioral Sciences*. Routledge.
- Davey, A. (2009). *Statistical Power Analysis with Missing Data: A Structural Equation Modeling Approach*. Routledge.
- Hedberg, E. C. (2017). *Introduction to Power Analysis: Two-Group Studies*. SAGE Publications.
- Liu, X. S. (2013). *Statistical Power Analysis for the Social and Behavioral Sciences: Basic and Advanced Techniques*. Routledge.
- Moerbeek, M., & Teerenstra, S. (2015). *Power Analysis of Trials with Multilevel Data*. CRC Press.
- Murphy, K. R., Myors, B., & Wolach, A. (2014). *Statistical Power Analysis: A Simple and General Model for Traditional and Modern Hypothesis Tests (Fourth Ed.)*. Routledge.
- Ryan, T. P. (2013). *Sample Size Determination and Power*. John Wiley & Sons.

Thank You!

Questions?